Sraffa’s Given Quantities of Output and Keynes’s Principle of Effective Demand

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Introduction

Sraffa (1960) takes the quantities of output as given when considering the determination of prices. Sraffa does not give any hint at how the quantities of output have come to be what they are. A commonly accepted interpretation of the given quantities of output in the Sraffa system is that they reflect the state (level and composition) of demand (see e.g. Garegnani, 1984, 1990; Kurz, 1990, 1992, 1994; Cesaratto, 1995). It is in that vein when the ‘Sraffian Keynesians’ attempt to synthesise Sraffa and Keynes; that is, to provide an integrated framework in which the Sraffian system serves for the determination of prices and the Keynesian principle of effective demand for the determination of the quantities of output. Some important work along these lines was done in the 1980s (e.g. Eatwell, 1983; papers in Eatwell and Milgate, 1983 and in Bharadwaj and Schefold, 1990), though it seems that the zeal has somehow petered out since the 1990s. An important exception is Serrano’s (1995) idea of ‘Sraffian multiplier’; however, it has brought about some criticisms within the Sraffian camp (Trezzini, 1995, 1998, Park, 2000) Palumbo (1996), Garegnani and Palumbo (1998), Palumbo and Trezzini (2003) provide the general directions for research along the lines concerned. Recently Trezzini (2005) and Garegnani and Trezzini (2010) pursue some particular ‘Sraffian Keynesian’ lines of research.

The following pages present another strand of attempt at synthesising Sraffa and Keynes, in particular in a formal framework. The framework consists of three systems of equations. The first is the price system *à la* Sraffa but, constant returns to scale assumed, represented in terms of the material input and labour input coefficients. Prices determined in this system are those which would emerge in a *fully-adjusted position* of the economy, that is, when productive capacity (the size and composition of the means of production) is fully adjusted to the state of demand so that it is utilised at the ‘normal’ level. The second is a variation of the usual Leontief quantity system where the economy is on a balanced growth. We take this quantity system as representing the ‘Golden Age Growth (GAG)’ state of the economy, where productive capacity in each industry is fully adjusted to demand so that it is utilised at the ‘normal’ level and *also* the labour force available in the economy as a whole are fully employed. The GAG state need be considered first, for this state will serve as the baseline with which the effective-demand-constrained (EDC) state of the economy is compared. The third system is one which is designed to analyse an economy in the EDC state where the quantities of output are determined in accordance with the state of effective demand. We shall represent the state of effective demand in terms of the volumes of (gross) investment in the *respective industries* of the economy, expressed in relative size to that which would emerge in the GAG state. As we are concerned with the long-period configuration of the economy, the EDC state we shall consider is a fully-adjusted position. The difference from the GAG state, which is by definition also characterised as a fully-adjusted position, is revealed in the *size* of the stock of means of production in each industry and, hence, in the level of capacity output and of capacity saving (Garegnani, 1962, 1982, 1983, 1992). Productive equipment of a size different from the GAG state being utilised at the normal level, the level of labour employment in the EDC state must be different from that in the GAG state, usually less than the level of full employment.

The price system

Consider an economy consisting of n single-product industries. The economy uses a technique represented by a semi-positive $n \times n$ material input coefficients matrix ($\mathbf{A}$) and a positive $n \times 1$ labour coefficients vector ($\mathbf{l}$).

The value of output produced in each industry consists of the value of replacement, profits and wages. This is expressed by the following Sraffa price system, with $\mathbf{p}$ standing for the $n \times 1$ vector of prices, r for the uniform rate of profits, w for the wage rate, and $\mathbf{d}$ for a given semi-positive $1 \times n$ vector of the unit basket of consumption goods:

$$(1+r)\mathbf{Ap} + w\mathbf{l} = \mathbf{p} \qquad (1a)$$

$$\mathbf{dp} = 1 \qquad (1b)$$

Equations (1a) represent the relationships which will be established when the means of production in each industry have been fully adjusted to the state of demand so that productive capacity in each industry is utilised at the 'normal' level—that is, when the economy is in a *fully-adjusted position* (Vianello, 1985). Equation (1b) normalises the prices of n commodities. Equations (1a) and (1b) together determine $\mathbf{p}$ and w for a given value of r.

The 'Golden Age Growth' state of the economy

Output produced in each industry is used as replacement, net investment and consumption. On the assumption that output grows at a uniform rate across the respective industries, the following variant of the Leontief quantity system holds:

$$(1+g^*)\mathbf{x}^*\mathbf{A} + h^*\mathbf{d} = \mathbf{x}^* \qquad (2a)$$

$$\mathbf{x}^*\mathbf{l} = 1 \qquad (2b)$$

where g^* = a uniform rate of output growth; h^* = the 'level' of consumption (the number of consumption baskets).[1]

Equation (2b) has two roles. One is to normalise the quantities of output. The economy

[1] Thus, it is being assumed that different income groups—wage earners and profit earners—consume the same unit bundle of commodities.

described in (2a) is a growing economy at the rate of g^*, so that the 'level' of output of the *i*-th industry, $x_i{}^*$, is not an 'absolute' level but a quantity defined *relative* to the 'levels' of output in the other industries. Another role is to express the condition of full employment of labour. The left-hand side of the equation stands for the amount of labour employed in the economy as a whole, and the right-hand side for the total labour force available in the economy, which is in turn normalised as unity (but the total labour force may be growing over time).

The economy which is described by equations (2a) is in a fully-adjusted position. The size and composition of the means of production have been fully adjusted to the level of output, so that $x_i{}^*$ is the level which will ensue from the normal utilisation of the productive capacity of the *i*-th industry. Thus, the system consisting of equations (2a) and (2b) describes an economy in which both the normal utilisation of productive capacity and the full employment of labour prevail. This is the 'Golden Age Growth (GAG)' state of the economy.

The price system and the quantity system are connected by the condition that investment and saving are equal *for the economy as a whole*. For the reasons to be made clear below, it is more convenient to consider investment and saving in the way as Kaldor (1966) does: in reference with corporations and households. Corporations retain, on average, a fraction s_f $(0 \leq s_f \leq 1)$ of their profits for net investment. Aggregate retained profits are $S_f \equiv s_f r \mathbf{x}^* \mathbf{Ap}$. For the portion of net investment that requires more than retained profits, corporations resort to the issue of new shares. The value of aggregate net investment that is financed through the new issue of shares is, following Kaldor, represented by $\phi^* g^* \mathbf{x}^* \mathbf{Ap}$, with $|\phi^*| < 1$. Corporations' plans of net investment and their financing in the economy as a whole are then represented by the following equation:

$$g^* \mathbf{x}^* \mathbf{Ap} = s_f r \mathbf{x}^* \mathbf{Ap} + \phi^* g^* \mathbf{x}^* \mathbf{Ap} \qquad (3)$$

which is reduced to a formulation that expresses the uniform net rate of growth as[2]

$$g^* = \frac{s_f r}{1 - \phi^*} \qquad (4)$$

Corporate shares are purchased by households. Households earn income from two sources: labour (which returns wages) and the holding of shares (which returns dividends and

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[2] In Kaldor (1966), equation (3) is used to determine the uniform rate of profits for a given behaviour of corporations (the rate of growth, the retention ratio and the external financing ratio). But in our setting the uniform rate of profits is exogenously given; as will be seen shortly, what is to be determined endogenously is the rate of growth and the external financing ratio, given the retention ratio in addition.

capital gain). We follow Kaldor's formulation of the behaviour of household saving: net saving out of wages is represented by a fraction s_w $(0 \le s_w \le 1)$ of their wages and net consumption out of the share holding by a fraction of c of capital gain (Γ).Thus, the total net saving of households is $S_h \equiv s_w w \mathbf{x}^* \mathbf{l} - c\Gamma$. Capital gain is calculated as $\Gamma = (v - \phi^*) g^* \mathbf{x}^* \mathbf{Ap}$ where v = valuation ratio (the ratio between the share market's valuation of the corporations in the economy and the replacement costs of their means of production). Equilibrium in the share market requires equality between the supply of and the demand for new shares:

$$S_h \equiv s_w w \mathbf{x}^* \mathbf{l} - c(v - \phi^*) g^* \mathbf{x}^* \mathbf{Ap} = \phi^* g^* \mathbf{x}^* \mathbf{Ap} \qquad (5)$$

Equation (5), together with equation (4), is used either, following Kaldor, to determine v for an autonomously determined value of ϕ^*, or, following his critics, to determine ϕ^* for $v = 1$. We take up the latter position: the share market's valuation of corporations at a value other than their replacement costs prompts (or forces) corporations to adjust their strategy of share issuance (probably because of the impacts that the share market's valuation exerts on the demand for shares). From equations (4) and (5) we can find the values of ϕ^* and g^* that will ensure $v = 1$. With g^* thus determined, one can then get x_i^* and h^* from (2a) and (2b).

Using the share market equilibrium condition (5), one can substitute household saving (the demand for shares) into the place of the new issue of shares (the supply of shares) in equation (3), thereby confirming the fulfilment of the macroeconomic equilibrium condition that aggregate investment is equal to aggregate saving. The volume of aggregate saving that will then appear in the right-hand side of equation (3) is one which will emerge from the normal utilisation of the stock of means of production and the full employment of labour, because x_i^* is the level of output which will emerge in those conditions. As the rate of profits *r* and the retention ratio of corporations s_f are exogenously given (and ϕ^*, g^* and x_i^*'s are fully determined in reference to them), the right-hand side is a known quantity once the former variables are known. Thus, this volume of saving is determined without any reference to the volume of investment; in other words, it is *prior to the volume of investment*. Equality between investment and saving, then, implies that it is the left-hand side (the volume of investment) that should adjust to the right-hand side (the volume of saving). As the right-hand side is the volume of saving that will accrue from the normal utilisation of productive capacity, it is called 'capacity saving'; accordingly, the volume of investment that should adjust to it is similarly called 'capacity investment', which is to be distinguished from *autonomous investment*.

The effective-demand-constrained state of the economy[3]

Contrary to the causality between investment and saving established in the GAG state, the Keynesian principle of effective demand asserts that investment has a causal priority over (capacity) saving and that saving should hence be adjusted to prior-given investment. We assume the position that the autonomy of investment is represented by the autonomously given volume of *gross* investment (in value terms) undertaken in each industry (and that the autonomously given volumes of gross investment in the respective industries are the manifestation of the state of effective demand).[4]

We consider an economy which is in a fully-adjusted position even though it is constrained by effective demand. Constrained by effective demand, output in the respective industries will be at the levels corresponding to the state of effective demand; in the long period, the size and composition of the means of production are fully adjusted to those levels of output, so as to produce them at the 'normal' degree of utilisation. Thus, prices are determined, as for the GAG state, in reference to the system (1a) and (1b).

Gross investment *in an individual industry* is financed through profits retained and new shares issued by corporations operating in that industry. Corporations in the *i*-th industry retain, on average, a fraction s_f of their profits and issue new shares in such a way that a fraction ϕ_i of their net investment is financed through new shares:[5]

$$J_i = (1 + s_f r) x_i \mathbf{A}_i \mathbf{p} + \phi_i (J_i - x_i \mathbf{A}_i \mathbf{p}) \tag{6}$$

The reading of equations (6) requires some caution. The GAG counterpart of equations (6), that is, equation (3), is read as expressing causality running from the right-hand side to the left-hand side; this direction of causality fits the scenario in which planned investment is realised (result) through its financing by means of retained profits and share issues (cause) which are rendered possible by the volume of saving that has been generated in accordance

3 Park (1994, 1997A, 2010), inspired by Garegnani (1962, 1982, 1983, 1992), discusses some background ideas for the argument contained in this section.

4 It may sound nonsensical to take a volume of investment as autonomously given in a long-period analysis, where quantities are supposed to change their size over time. This concern shall be addressed later. Also, it is more appropriate to consider *gross*, not net, investment as representing the state of effective demand; as the reason shall be made apparent below (see footnote **), this is particularly so for the long period, in which the size and composition of the means of production are endogenously determined.

5 For the GAG state, no argument was carried out at the individual industry level. But if we add the assumption that the average retention ratio of each industry is uniform at s_f to the already-mentioned assumption that the rate of growth is uniform, then it follows that, as is seen in equation (4), that the fraction of new investment financed through the new issue of shares is uniform at ϕ^* across the industries. It is to be noted that, in the EDC state, the average retention ratio of each industry need not be uniform, though we assume the uniformity for simplification.

with the normal utilisation of productive capacity and the full employment of labour. However, the causality to be read out from equations (6) must be to the contrary: the left-hand side of equations (6), being an autonomously given quantity of investment, has a causal priority. The right-hand side must be a result: a quantity of output in an individual industry is generated in accordance with the autonomous investment made in that industry (and possibly in accordance with investment made in other industries as well) and corporations' execution of investment is consolidated by retained profits and the issue of new shares, these latter being determined in reference to the level of output of the industry.

There is a point that deserves emphasis regarding equations (6). Equations (6) should not be read as meaning that the volume of saving that is generated in an individual industry due to the investment made in that industry is equal to the volume of saving that consolidates that investment in the final event; the right-hand side of equation (6) is the latter volume of saving. The equality, though holding for the economy as a whole, cannot generally be said of individual industries. The reason is that, whilst corporations in an industry make saving with the specific purpose of investing in that very own industry, households save with no such specific usage in mind (see Keynes, 1936, ch. 12): household saving is to be considered, at least initially, at the level of the economy as a whole. It is through the working of the share market that aggregate household saving is allocated to the respective industries: household saving *flows into* each of the industries in the form of the demand for shares supplied by the respective industries.[6] It is through household saving that corporations' plan of financing their investment externally (the supply of shares) is eventually consolidated, household saving having already been generated through investment.[7]

From equation (6) one can immediately obtain, for given J_i,[8]

[6] It is Kaldor's (1966) formulation that shows this mechanism—the mechanism of allocating aggregate saving to individual industries—explicit, and this—together with our necessity of considering investment at the industry level—is the very reason why we adopt it. If the counterpart of equations (6) is to be formulated in the way of Kaldor (1956) or Pasinetti (1960), explicit consideration must be made of how to incorporate such mechanism in the formulation; see footnote 10 below.

[7] Corporations may initially finance their investment (exceeding the quantity supported by retained profits) through short-term bank loans and then eventually consolidate their financing through the issuance of long-term debts such as bonds or shares.

[8] Had we represented the autonomy of investment in terms of *net* investment, we would have had a similar result, $x_i = \frac{(1-\phi_i)I_i}{s_f r \mathbf{A}_i \mathbf{p}}$, with I_i expressing the volume of net investment. Then, in a stationary state of the economy, where $I_i = 0$, we would have $x_i = 0$ (if r is not zero) or x_i indeterminate (if r is zero). Both cases yield an absurd result. In contrast, expressing the autonomy of investment in terms of *gross* investment, we shall always have $J_i > 0$ as far as the i-th industry exists; hence, guaranteeing $x_i > 0$ for a stationary economy (at least for a finite value of r).

$$x_i = \frac{(1-\phi_i)J_i}{(1+s_f r-\phi_i)\mathbf{A}_i\mathbf{p}} \tag{7}$$

These equations should not be taken as 'determining' the levels of output in reference to the variables appearing in the right hand side, for ϕ_i is yet to be determined.[9] The main role of equations (6) is that of expressing the *constraint of effective demand* for each of the industries: the volume of gross investment is autonomously given and the economy must adjust itself in accordance with this constraint, the adjustment taking the form of appropriate variations in the level of output (x_i) and the external financing ratio (ϕ_i) (variations in the latter variable will affect its counterparts of the other industries and hence the levels of output of the other industries).

There being no *a priori* reason that the external financing ratios be uniform across the industries, the net rate of growth is, in general, different across the industries:

$$g_i \equiv \frac{J_i}{x_i\mathbf{A}_i\mathbf{p}} - 1 = \frac{s_f r}{1-\phi_i} = \left(\frac{1-\phi^*}{1-\phi_i}\right) g^* \tag{8}$$

The last equality comes from equation (4). One sees that $g_i > g^*$ if $\phi_i > \phi^*$.

As in the GAG state, the market for each commodity must clear in a long-period position. The quantity produced of each commodity is used for replacement, net investment or consumption, and these three usages of the commodity exhaust the quantity produced thereof in equilibrium. Hence, the following relationships, the EDC counterpart of equations (2a), must hold:

$$\mathbf{x}(\mathbf{I}+\hat{\mathbf{g}})\mathbf{A} + h\mathbf{d} = \mathbf{x} \tag{9}$$

where $\mathbf{I}$ is the $n \times n$ identity matrix and $\hat{\mathbf{g}}$ is the diagonal matrix generated from the

[9] If for the moment we assume the exogeneity of ϕ_i *à la* Kaldor (1966), then one can carry out some comparative statics exercises directly with equations (7). For example, x_i moves in step with J_i; it follows that the volume of aggregate output moves also in step with the volume of gross investment in any industry. It is thus implied that the aggregate volume of (gross) capacity saving moves in the same direction as (gross) investment in any industry (and by the same quantity as the change in aggregate gross investment). The paradox of thrift is also observed: a rise in the propensity to save (s_f) lowers the level of output. One can also observe what can be called the 'paradox of external financing'. If a greater fraction of net investment is financed through new issue of shares whilst the quantity of gross investment remains unchanged (a ceteris paribus rise in ϕ_i), the level of output in the own industry is lowered.

g_i's.[10]

Post-multiply equations (9) by $\mathbf{p}$; then, using the definition of aggregate net saving (*S*) as the difference between the value of aggregate gross output ($\mathbf{xp}$) and the value of aggregate consumption ($h\mathbf{dp}$), one gets the equality between investment and saving in aggregate terms. Aggregate saving consists of aggregate gross corporate saving (gross retained profits, $S_f = \sum(1+s_f)x_i\mathbf{A}_i\mathbf{p}$) and aggregate household saving (S_h):

$$S \equiv \mathbf{xp} - h\mathbf{dp} = \mathbf{x}(\mathbf{I}+\hat{\mathbf{g}})\mathbf{Ap} = \sum J_i = S_f + S_h \qquad (10)$$

Sum equations (6) over the industries; then, use equation (10) to get

$$S_h = \sum \phi_i (J_i - x_i \mathbf{A}_i \mathbf{p})$$

This is precisely the equilibrium condition for the share market. Following Kaldor's formulation for household saving and applying a long-period condition that the valuation ratio in each industry be unity, this condition is expressed as

$$s_w w \sum x_i l_i - c \sum (1-\phi_i) g_i x_i \mathbf{A}_i \mathbf{p} = \sum \phi_i g_i x_i \mathbf{A}_i \mathbf{p} \qquad (11)$$

The equation system for an EDC state consists of equations (6), (8), (9) and (11).[11] We

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[10] If the composition of the consumption basket ($\mathbf{d}$) is constant over time, this means that the consumption of each consumption good grows at a uniform rate. But note that g_i is the rate of growth defined for each industry, not for each commodity, with the result that different kinds of commodities may well grow at different rates. The rates of growth of each commodity (γ_j), the industrial rates of growth (g_i) and the rates of growth of consumption of each consumption good (ξ_i) must have the following relationships:

$$\gamma_j x_j = \sum_{i=1}^{n} (1+g_i)\gamma_i x_i a_{ij} + \xi_j (hd_j)$$

ξ_i's are given exogenously (by the 'preference' of consumers) and g_i's are determined in the system described in the text; thus, one can determine γ_i's in reference to the above relationships.

[11] There can be alternative formulations, in particular, regarding the relationship between investment and saving at the industry level. One such alternative, which considers saving behaviour in terms of income categories *à la* Kaldor (1956) (and, by extension, in terms of social classes *à la* Pasinetti (1960)), is as follows. Autonomous (gross) investment in an individual industry contributes to the generation of output in that industry; then, we can calculate the volume of saving that is generated thanks to that output. Assuming for simplicity that the saving propensity out of wages is on average zero for each industry, the gross saving generated out of the output of the *i*-th industry is $(1+s_r r)x_i\mathbf{A}_i\mathbf{p}$ where s_r is

have $3n+1$ unknowns (x_i's, g_i's, ϕ_i's and *h*) in the same number of independent equations. The system fully reflects the working of the principle of effective demand. The volume of autonomous (gross) investment, J_i, either singly for an individual industry or jointly with its counterparts in some or all of the other industries, will affect the economy's configuration of x_i's, g_i's, ϕ_i's and *h*. The resulting quantity of output in each industry (x_i) is at the normal level, ensuing from the fully-adjusted stock of means of production ($x_i\mathbf{A}_i$) in that industry; hence, the saving is the *capacity saving* ensuing from that stock of productive equipment. The last equality in equation (10) shows that, in the aggregate, investment generates saving of the same volume as the investment. Thus, regarding investment and saving, causality runs in the opposite direction to the case of the GAG state of the economy. (Gross) investment is a given quantity, and therefore it is the capacity saving that should adjust. This adjustment of capacity saving to investment is rendered possible through the adjustment of the size of the stock of means of production in accordance with a given state of effective demand, that is, given volumes of gross investment. The aggregate capacity saving thus generated is channelled into each industry through the process of consolidating investment in each industry—in the form of retained profits and the purchase of corporative shares for each individual industry (hence, at the individual industry level as well as at the level of the economy as a whole, matching the capacity saving with the autonomously given investment by the same volume). This is indeed the main feature of the long-period, in contrast with the short-period, manifestation of the principle of effective demand. In the long period, it is (mainly) through variations in the size of productive capacity that independent investment generates the corresponding volume of capacity saving (Garegnani, 1962, 1982,

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the saving propensity out of profits averaged over the industry (and we assume that this saving propensity is uniform across the industries). But in general the volume of saving thus generated will be different from the volume of saving that will support (consolidate) investment in that industry. For, as has been mentioned in the text, profit-earners make saving with no specific usage in mind, and this aggregate saving is allocated to individual industries through some economic mechanism(s), such as operations in the share market, the bond market and/or the loan market. The result is that the volume of saving that has finally found its place in a particular individual industry (to consolidate the investment made in that industry) will in general be larger or smaller than the volume of saving that has been generated due to the investment made in that industry. In a sense, there will in general be, in net terms, an 'inflow' or an 'outflow' of saving into or out of individual industries. One may denote such inter-industry flows of saving by $\delta_i x_i \mathbf{Ap}$ (there may be many alternative formulations for this): $\delta_i > 0$ means a net *inflow* of saving into the *i*-th industry above the volume of saving that is generated thanks to the output in that industry; $\delta_i < 0$ means a net *outflow* of saving out of the *i*-th industry so that the volume of saving that will be used in supporting the investment in the industry is smaller than the volume of saving that has been generated in association with the output in the industry. Then, the counterpart of equation (6) will be $J_i = (1+s_r r)x_i\mathbf{A}_i\mathbf{p} + \delta_i x_i \mathbf{A}_i\mathbf{p}$. The aggregate result of these inter-industry flows of saving is, of course, *nil*: hence, we have, as the counterpart of (11), $\sum \delta_i x_i \mathbf{A}_i \mathbf{p} = 0$.

1983, 1990).[12]

Up to now, we have been talking about J_i as if standing for a certain 'absolute' volume of investment. As we have discussed for the GAG state, this, if literally understood, makes little sense for a growing economy. Our intention is to analyse effects of the constraint of effective demand in the long period. The state in which effective demand is *not* a constraining factor is construed as the GAG state, in the sense that the volume of gross investment carried out, $J_i^* \equiv (1+g^*)x_i^* \mathbf{A}_i\mathbf{p}$, is that which is *required* (thus, not autonomous) for continuous normal utilisation of productive capacity and continuous full employment of labour, and this solely in reference to the private sector (without the intervention of the government). (Of course, J_i^* is itself to be understood not as an absolute volume but as a volume that grows over time at the rate of g^*). Then, the state of effective demand is better understood *in terms of comparison with the GAG state*. The volume of gross investment undertaken in each industry in the EDC state (J_i) is to be understood relative to J_i^*. Let us denote by z_i the ratio that J_i bears to J_i^*:[13]

$$z_i \equiv J_i / J_i^* \tag{12}$$

If z_i is lower than otherwise, it means that the state of effective demand in the *i*-th industry is relatively for the worse; conversely, if z_i is higher. The GAG state of the economy, with no constraint of effective demand, is represented by the condition that $z_i = 1$ for all *i*.

It immediately follows from (4) and (8) that

$$\frac{x_i}{x_i^*} \equiv \chi_i = \frac{J_i/(1+g_i)\mathbf{A}_i\mathbf{p}}{J_i^*/(1+g^*)\mathbf{A}_i\mathbf{p}} = \left(\frac{1+g^*}{1+g_i}\right) z_i = \Phi_i z_i \tag{13}$$

where $\Phi_i \equiv \dfrac{(1-\phi_i)(1+s_f r-\phi^*)}{(1-\phi^*)(1+s_f r-\phi_i)} > 0$. The higher z_i is ceteris paribus, the higher will be the

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[12] There are other routes through which autonomous investment generates the corresponding amount of saving. One is that identified by the Cambridge school with Kaldor, Pasinetti and Robinson as its representatives: variations in the normal state of income distribution (changes in *r* and *w*). Another is that emphasised by the Kaleckians such as Steindl (1952), Rowthorn (1981) and Dutt (1990): variations in the (long-period equilibrium) degree of utilisation of productive capacity (which in turn implies changes in the long-period equilibrium levels of *r* whilst the wage rate may remain unchanged). These routes have been under criticisms (Garegnani, 1982, 1983, 1992; Ciccone, 1984, 1986; Committeri, 1986; Park, 1997B, 2010).

[13] One may make z_i a function of some relevant endogenous variables; for example, $z_i = z_i(r)$, $dz_i/dr > 0$. But this is a secondary, if important, matter.

ratio that the EDC level of output bears to the GAG counterpart.[14]

A long-period analysis of the EDC state requires consideration of some more constraints beside the constraint of effective demand. These constraints can eventually be expressed as those to be imposed on the 'behavioural variable' of our main concern: z_i's. First of all, recall that the economy-wide volume of gross investment for the GAG state is that which will maintain the normal utilisation of productive capacity in the economy as a whole (and also in each industry of the economy). We take this volume of gross investment as the baseline with which to compare the economy-wide volume of gross investment made in the EDC state. Thus, it will be the case in general that unless there is a spur from outside (that is, outside of the private sector, such as the government or the foreign sector), the total volume of investment undertaken in the EDC state does not exceed that in the GAG state:

$$\sum_{i=1}^{n} J_i \leq \sum_{i=1}^{n} J_i^*$$

or, using (12),

$$\sum_{i=1}^{n} z_i J_i^* \leq \sum_{i=1}^{n} J_i^* \qquad (14)$$

This constraint will be called the *constraint of normal utilisation* (NU). If constraint (14) holds in strict inequality, this means that the economy-wide state of effective demand is insufficient for the normal utilisation of productive capacity in the respective industries.

Whether constraint (13) is in operation either as equality or as strict inequality, however, it may be the case that the volume of investment in some individual industries can be larger than that in the GAG state. This is because a greater volume of investment will make its way into those industries in which 'animal spirits' are higher. Industries in the GAG state utilise their productive capacity at the normal level and this is represented by $z_i = 1$. If autonomous investment in an individual industry is such that $z_i > 1$, this means that productive capacity in that industry will be stretched to be utilised *above* the normal level. This 'over'-utilisation is possible up to a certain limit, for the decision on the normal degree of utilisation of productive capacity usually takes account of some margin for 'over'-utilisation (Steindl, 1956). If we denote that limit of 'over'-utilisation by $\bar{z}_i$, then z_i must observe the following condition:

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[14] This ratio is affected by the uniform rate of profits, the direction of influence being dependent of the sign of $\phi^* - \phi_i$: if an industry relies more on external financing than the GAG case ($\phi_i > \phi^*$), the ratio moves inversely with *r* (but recall that a change in r is accompanied in general by a change in ϕ_i).

$$0 < z_i \le \overline{z}_i \quad \text{with} \quad \overline{z}_i > 1 \tag{15}$$

We may call this the *constraint of full utilisation* (FU), using the expression 'full' as meaning the physical limit of utilisation of productive capacity. This constraint is specified for each industry; however, combined with the constraint of NU above, it must be the case that $1 < z_i \le \overline{z}_i$ for some industries implies $0 < z_j < 1$ for some other industries.

We may interpret the full employment of labour in its strict meaning as the state in which there is no further labour force available. Then, it must be the case that the total employment in the economy as a whole cannot exceed that obtained in the GAG state, for the GAG state employs all the available labour. This is the *constraint of full employment of labour* (FE).

$$\sum_{i=1}^{n} x_i l_i \le 1$$

or using (12),

$$\sum_{i=1}^{n} z_i \Phi_i x_i{}^* l_i \le 1 \tag{16}$$

Of course, one can adopt a less strict interpretation of full employment, such that an increase in the participation rate, accompanying an economy-wide increase in investment, allows for the total amount of labour employed in the economy to be larger than unity (up to a certain limit). But our purpose is sufficiently served by the strict interpretation.

Sraffa (1960, p. 5, fn. 1) explicitly states that his 'formulation [for the determination of prices] presupposes the system's being in a self-replacing state': for each commodity, the aggregate quantity produced in the economy should not be less than the aggregate quantity used up as the means of production across the industries:

$$x_j - \sum_{i=1}^{n} x_i a_{ij} \ge 0$$

or using (12)

$$\Phi_j z_j x_j{}^* - \sum_{i=1}^{n} \Phi_i z_i x_i{}^* a_{ij} \ge 0 \tag{17}$$

where a_{ij} is the *ij*-th element of **A**. This is the *constraint of self-replacing state* (SR). If constraint (17) holds as equality for all *j*, then the economy is one without any surplus; if strict inequality holds for at least one industry, then the economy is a surplus economy.

In equilibrium, ϕ_i's are determined in reference to z_i's and the 'fundamental variables' (the technology, the rate of profits and the saving or consumption propensities): $\phi_i = \phi_i(z_1, z_2, \cdots, z_n; \mathbf{A}, \mathbf{l}, r, s_f, c)$. Now, it must be the case that $|\phi_i| < 1$ for all *i*. This implies that there may be some constraint to be imposed on the combinations of z_i's that will keep the equilibrium values of ϕ_i within the permitted range:

$$(z_1, z_2, \cdots, z_n) \in \{(z_1, z_2, \cdots, z_n) : -1 < \phi_i(z_1, z_2, \cdots, z_n) < 1, \ \forall i\} \qquad (18)$$

This constraint will be christened the *constraint of financial market* (FM).

Those combinations of z_i's which satisfy all the constraints of NU, FU, FE, SR and FM will define the set of possible long-period positions in the EDC state. A particular combination of z_i's among them will be called a '*point of effective demand*' (PED). A PED satisfies the share market equilibrium (11). Expressed in terms of z_i, condition (11) is transformed into

$$s_w w \sum z_i \Phi_i x_i * l_i - c s_f r \sum z_i \Phi_i x_i * \mathbf{A}_i \mathbf{p} = s_f r \sum z_i \left(\frac{\phi_i}{1-\phi_i} \right) \Phi_i x_i * \mathbf{A}_i \mathbf{p} \qquad (19)$$

Thus, we shall call equation (18), with all the endogenous variables having been determined, the *equation of long-period positions* (LP).

An example of a two-industry economy

Consider an economy which consists of two industries, one producing the means of production ('machine', commodity 1) and the other producing the consumption good ('corn', commodity 2); the machine is a circulating capital so that it is used up in a unit production period. Thus, the technique in use is represented by the following:

$$\mathbf{A} = \begin{pmatrix} a_{11} & 0 \\ a_{21} & 0 \end{pmatrix}, \ 0 < a_{ij} < 1;$$

$$\mathbf{l} = \begin{pmatrix} l_1 \\ l_2 \end{pmatrix}, \quad l_i > 0$$

The unit basket of consumption is $\mathbf{d} = (0 \quad 1)$; thus, $p_2 = 1$. For a given level of r, the system of (1.a) and (1.b) determines *w* and p_1. In accordance with the saving behaviours of corporations and households and with the condition that the valuation ratio is unity in all the industries, the system consisting of (2a), (2b) and (3) determines the GAG levels of output and the normal rate of accumulation: x_1^*, x_2^* and g^*. The volume of capacity investment in an individual industry is $J_i^* = (1+g^*)x_i^*\mathbf{A}_i\mathbf{p}$.

The EDC state will observe relationships (6) , (8), (9) and (11) for i =1, 2, with $\mathbf{A}$ and $\mathbf{l}$ as above. The autonomous volume of gross investment in an individual industry bears a ratio z_i to J_i^*. Then, the five constraints that z_i's must satisfy are as follows:

(i) normal utilisation (NU):

$$z_2 \le \left(1 + \frac{x_1^*\mathbf{A}_1\mathbf{p}}{x_2^*\mathbf{A}_2\mathbf{p}}\right) - z_1\left(\frac{x_1^*}{x_2^*}\right)\left(\frac{\mathbf{A}_1\mathbf{p}}{\mathbf{A}_2\mathbf{p}}\right) \tag{14'}$$

(ii) full utilisation (FU):

$$0 < z_i \le \overline{z}_i, \quad i = 1,2 \tag{15'}$$

(iii) full employment (FE):

$$z_2 \le \left(\frac{1}{x_2^* l_2 \Phi_2}\right) - z_1\left(\frac{x_1^*}{x_2^*}\right)\left(\frac{\Phi_1}{\Phi_2}\right)\left(\frac{l_1}{l_2}\right) \tag{16'}$$

(iv) self-replacing state (SR):

$$z_2 \le z_1\left(\frac{x_1^*}{x_2^*}\right)\left(\frac{\Phi_1}{\Phi_2}\right)\left(\frac{1-a_{11}}{a_{21}}\right) \tag{17'}$$

(v) financial market (FM)

$$z_2 = z_1\left(\frac{x_1^*}{x_2^*}\right)\left(\frac{\Phi_1}{\Phi_2}\right)\left(-\frac{s_w w l_1 - s_f r(c + \frac{\phi_1}{1-\phi_1})\mathbf{A}_1\mathbf{p}}{s_w w l_2 - s_f r(c + \frac{\phi_2}{1-\phi_2})\mathbf{A}_2\mathbf{p}}\right) \equiv \zeta z_2 \tag{19'}$$

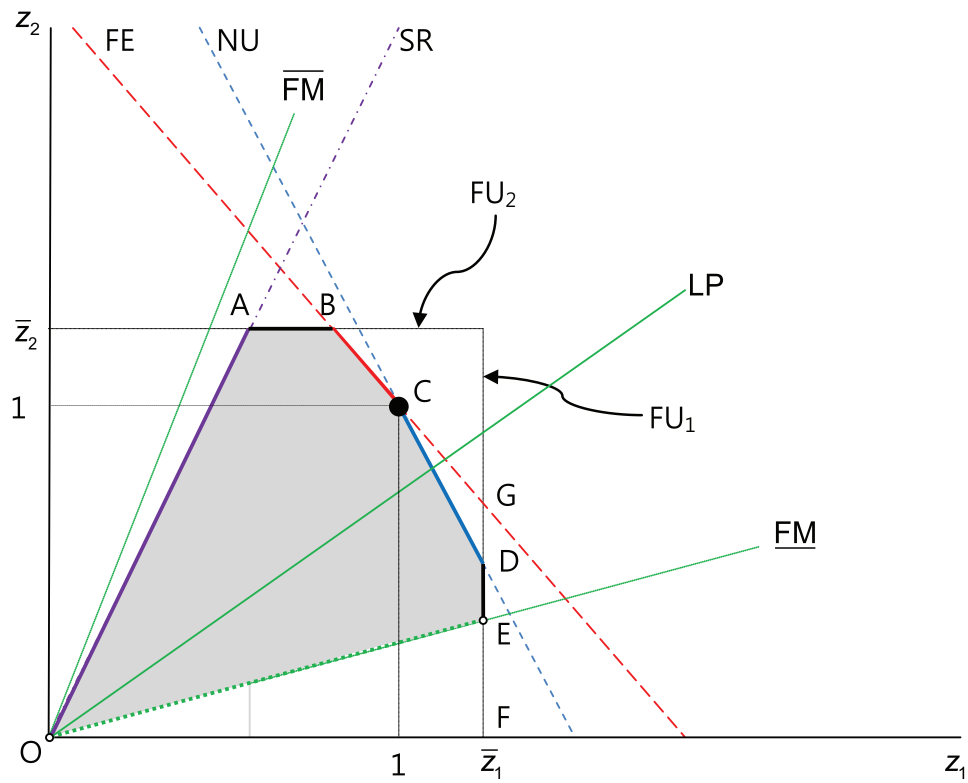
with

$$\max(0, \underline{\zeta}) \le \zeta \le \min(\overline{\zeta}, \infty) \tag{18'}$$

where $\underline{\zeta}$ is the infimum of ζ and $\overline{\zeta}$ the supremum of ζ for $|\phi_i| < 1$ for all i (strict inequality applies if $\underline{\zeta}$ and $\overline{\zeta}$ are respectively the case).

The following figure, which incorporates the five constraints, can serve as an aid in discussing some issues regarding effective demand in the long-period.[15]

[15] The figure is drawn with z_i's as the variables for the two axes. For expositing certain aspects of an EDC state, it may be more convenient to draw the figure in reference to the ratio of the EDC level of output to its GAG counterpart for each industry ($\chi_i \equiv x_i / x_i^*$). But we leave the exercise to the reader.



Long period positions that are possible in an EDC state are represented by the combinations of z_1 and z_2 constituting the inside and the boundary (except segment OE) of polygon OABCDE (a 'Diamond'). We have drawn the lines such that line $\overline{\text{FM}}$, which corresponds to $\zeta = \overline{\zeta} < \infty$, lies above line SR and line $\underline{\text{FM}}$, which corresponds to $\zeta = \underline{\zeta} > 0$, cuts the internal part of segment DF. The positions of lines FE, SR, $\overline{\text{FM}}$ and $\underline{\text{FM}}$ will change with z_1 and z_2. A point on line LP represents an exogenously given combination of z_1 and z_2 that satisfy all the constraints and also equation (19') with all the endogenous variables determined in appropriate ranges. Such a point is, thus, a PED. It immediately turns out that line LP is the set of PEDs which will result when z_1 and z_2 maintain a constant ratio (for this, the endogenous variables must adjust, with the result that FE, SR, $\overline{\text{FM}}$ and $\underline{\text{FM}}$ change their slopes).

Point C represents the GAG state, where $z_1 = z_2 = 1$: productive capacity is utilised at the normal level both in the economy as a whole and in each industry and the available labour force is fully employed (hence, at the intersection of lines NU and FE, which line LP goes through). The figure depicts an economy where the GAG state is also a self-replacing state (below line SR). It is possible for point C to line above the SR line; if so, the GAG state is a non-self-replacing state even though the technique in use allows the economy to be viable.[16] The name of the 'Golden Age Growth' state for point C follows the same christening of such a state of the economy by Joan Robinson (1962): 'starting with near full employment and a composition of the stock of plant appropriate to the desired rate of accumulation, near full employment is maintained' (p. 52).

The figure helps identify several causes for labour unemployment over the long period. The first refers to the case related to the area above line SR. The levels of gross investment in the respective industries should not go beyond segment OA; if they did, the economy would not be in a self-replacing state. But the full employment of labour requires investment to go beyond the OA segment. Unemployment of this type can exist even if the constraint of NU is not binding; that is, even if the economy is capable of investing and willing to invest at the level that is more than enough to attain the full employment of labour, this latter objective cannot be realised in the long period.

The second and the third are due to the constraint of the full utilisation of productive capacity in the respective industries, represented by segments AB and DE. No investment can be made, even if it is desired, if this investment requires the utilisation of productive capacity beyond the full level, for the full degree of utilisation is the maximum of physically possible levels of utilisation. In the figure, the full employment of labour requires investment in the corn industry above $\bar{z}_2$ (segment AB); this is physically impossible (and this is so even if both the willingness to invest and the capability of investment are more than enough to achieve full employment). This case is reminiscent of Joan Robinson's 'Creeping Platinum Age', in which 'the ratio of basic plant is too high' (Robinson, p. 57), that is, the machine-producing industry is too large relative to the corn-producing industry so that, to maintain the long-period configuration of the economy (in particular, a self-replacing state), productive capacity in the latter industry has to be stretched to be utilised up to its physical limit, with the result, however, that even this over-utilisation is not sufficient to guarantee full employment. Another case is one in which the full employment of labour could be attained if the utilisation of productive capacity of the machine industry could go beyond $\bar{z}_1$ (segment DE); however, again, this is physically impossible (and in the particular case of the figure, effective demand at the normal level, too, is not sufficient to guarantee full employment). This case corresponds to Robinson's 'Galloping Platinum Age', in which '"animal spirits" are high, and a large mass of non-employed labour is available, but the desired rate of growth cannot be attained because of lack of basic plant to produce plant' (Robinson, p. 56): the utilisation of productive capacity in the machine-producing industry is stretched up to the limit but with no success in attaining full employment. Unemployment of this type can exist even though 'animal spirits' are higher than enough to guarantee full employment, as is the case if the relative slope of lines FE and NU is reversed.

The fourth case is, perhaps, what can be appropriately called 'long-period structural

[16] For the distinction between a viable economy and a self-replacing economy, see Sraffa (1960, p. 5) and Chiodi (2002).

unemployment' due to the general lack of effective demand. Along segment CD, effective demand is just enough to guarantee continuous normal utilisation of productive capacity for the economy as a whole; however, even this level of overall effective demand, except at point C, is not sufficient to attain the full employment of labour. The reason is that the total volume of investment has been 'mis-allocated' among the industries, so that the composition of the aggregate output is not appropriate to bring about the full employment of labour, given the technique in use. This is what Robinson calls the 'Limping Golden Age', in which 'a steady rate of accumulation of capital may take place below full employment' (1960, p. 53), which would eventually settle at the 'Leaden Age' in which 'Malthusian misery checks the rate of growth of population … [so that] the rate of accumulation and the rate of growth of the labour force [are] equal, the ratio of non-employment being great enough to keep the latter down to equality with the former' (p. 54).

The last case is one that can be explained in reference to segment OE (with the relative slope of FE and NU reversed). Given the 'fundamentals' (the technique in use, the rate of profits and the saving propensities), it is impossible to achieve full employment even if 'animal spirits' are high enough. For whilst full employment requires an appropriate adjustment in the behaviour of corporations as regards their plans for external financing, the share (more generally, financial) market is not capable of accommodating the required adjustment. Full employment is possible only with changes in the 'fundamentals', which will enhance the capability of the financial market in this regard by removing the constraint expressed by line $\underline{\mathrm{FM}}$ (similar remarks can be made if $\overline{\mathrm{FM}}$ is below SR).

Only if LP cuts through segment BC can full employment be attained when the state of effective demand is high enough to guarantee it; this may happen even if the economy-wide level of effective demand is *lower* than that corresponding to the GAG state (shift the NU line to the left until it passes through point B; then, the general level of effective demand is lower but full employment is attained). However, the economy is restrained in another sense: restrained by the lack of further available labour even though 'animal spirits' can be higher than the level that corresponds to full employment. Robinson would describe this situation as the 'Restrained Golden Age', where '[w]ith a stock of plant appropriate to the desired rate of accumulation (which exceeds the rate of growth of population) and full employment already attained, the desired rate of accumulation cannot be realised, because the rate of growth of output per head … is not sufficient to make it possible' (p. 54).

With effective demand constraining the performance of the economy, however, the most usual state in which the economy finds itself will be well *inside* the Diamond (and on line LP): effective demand is such that*, in any industry*, *neither* continuous full employment of labour *nor* continuous normal utilisation of productive capacity is attained. However, in the long period, the economy will eventually settle at a fully-adjusted position with the stock of means of production fully adjusted to the state of demand so that it is utilised at the normal level. The state of effective demand exerts its effect *in the course of such full adjustment*, that is, in the form of the below-normal utilisation of productive capacity, let alone the unemployment of labour, while the economy goes through the process of adjustment. A final result of this process is a smaller size of productive capacity and thus a lower level of output at the normal utilisation of that productive capacity.

Suppose that the government—which we have not taken into consideration until now—tries, by increasing (or initiating) government expenditure, to help the economy to attain the full employment of labour.[17] An increase in government expenditure will be represented by a movement of PED (and a change in the slope of LP); the precise direction of its movement dependent, among others, on the allocation of the expenditure among the products of the respective industries (one may call the government decisions of this kind *industrial policy*).[18] Increase in government expenditure can achieve full employment only in the case of Limping or Restrained Golden Age, that is, only if the economy finds itself on a point of segment BCG (with government expenditure, effective demand in the economy as a whole can be larger than the limit that the private sector is subject to; hence, the part CG is now capable of being attained). This implies that mere increase in government expenditure, however large, may not always secure the full employment of labour unless the response of the private sector is such as to guarantee that line LP cuts segment BCG. But the adjustment of ϕ_i's in the private sector in response to the increase in government expenditure *and* its allocation among the commodities affects the slopes of the lines representing constraints FE, SR, $\overline{FM}$ and $\underline{FM}$ in addition to the slope of line LP. The success of fiscal policy (in the sense of increase in government expenditure) in its aim to attain the full employment of labour requires the company of appropriate *industrial policy* (in the sense of allocating government expenditure among the industries).

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[17] See Appendix.

[18] But the natural direction of movement will exclude South-West (output does not decrease in all the industries).

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Appendix: Introducing government expenditure

With the introduction of government expenditure, we need additionally consider taxation and the financing of budget deficit if any. Equation (1a) is to be modified, income tax being considered, to

$$[1+(1+t_r)r]\mathbf{Ap}+(1+t_w)w\mathbf{l}=\mathbf{p} \tag{A.1}$$

where t_r = tax rate on profits and t_w = tax rate on wages.

The counterpart of equation (6) is

$$J_i=[1+s_f(1-t_r)r]x_i\mathbf{A}_i\mathbf{p}+\phi_i(J_i-x_i\mathbf{Ap}) \tag{A.2}$$

from which one gets equations very similar to (7) and (8) respectively:

$$x_i=\frac{(1-\phi_i)J_i}{[1+s_f(1-t_r)r-\phi_i]\mathbf{A}_i\mathbf{p}} \tag{A.3}$$

$$g_i=\frac{s_f r(1-t_r)}{1-\phi_i} \tag{A.4}$$

Government expenditure takes the form of purchasing the various commodities produced in the economy. There are several possible ways of expressing it in the model. One such way is as follows. We assume that budget deficit is financed through T-bonds (public debt) only and T-bonds are perpetuals (consols). The government makes two kinds of decisions in relation to government expenditure: the total volume of expenditure and its allocation among the products of the respective industries. One may call the former decision (proper) *fiscal policy* and the latter *industrial policy*. As regards fiscal policy, the government tries to maintain a certain ratio between public debt and GNP (γ). Public debt is calculated as the total government expenditure (G) plus the interest payment by the government on T-bonds (ρB_{-1}, where B_{-1} is the value of the previously issued T-bonds and ρ is the rate of interest on T-bonds) minus the total tax revenue:

$$G=\gamma\mathbf{xp}+T-\rho B_{-1} \tag{A.5}$$

The government allocates G for purchasing the product of the *j*-th industry by the amount of

$$G_j=\alpha_j G=\alpha_j(\gamma\mathbf{xp}+T-\rho B_{-1}),\ \text{with}\ \sum\alpha_i=1 \tag{A.6}$$

Then, the physical quantity of the j-th commodity purchased by the government is

$$u_j = \alpha_j G / p_j \tag{A.7}$$

The market-clearing condition for each commodity requires the following relationships, which is an extension of equations (9):

$$\mathbf{x}(\mathbf{I} + \hat{\mathbf{g}})\mathbf{A} + h\mathbf{d} + \mathbf{u} = \mathbf{x} \tag{A.8}$$

where $\mathbf{u} = (u_1\ u_2 \cdots u_n)$.

Interest payment on T-bonds becomes part of the income of the private sector which purchases the bonds, and tax is levied on the interest income; if the tax rate on the interest income is t_B, the total tax revenue is

$$T = t_r r\mathbf{xAp} + t_w w\mathbf{xl} + t_B \rho B_{-1} \tag{A.9}$$

It is assumed for simplicity that T-bonds are purchased by households only.

Households have three sources of income: labour, the holding of corporate shares and the holding of T-bonds. For simplicity and in analogy with the formulation of household saving in the main text, we assume that saving out of wages is represented by the saving propensity s_w and consumption out of the total of property income by the fraction c of the sum of capital gain and the after-tax interest income; thus, the after-tax household saving is

$$\tilde{S}_h = s_w w(1 - t_w)\mathbf{xl} - c\left[\sum (1 - \phi_i) g_i x_i \mathbf{A}_i \mathbf{p} + (1 - t_B)\rho B_{-1}\right] \tag{A.10}$$

Households use this saving for two purposes: purchasing corporate shares and purchasing T-bonds. If the household portfolio decision is represented by η that is a fraction of the household saving which is used for purchasing corporate bonds, we have the following two equilibrium conditions, one for the share market and the other for the T-bond market:

$$\eta \tilde{S}_h = \sum \phi_i (J_i - x_i \mathbf{A}_i \mathbf{p}) \tag{A.11}$$

$$(1 - \eta)\tilde{S}_h = \gamma \mathbf{xp} \tag{A.12}$$

The part of the household saving which is used in purchasing corporate shares is sorted out into the respective industries in accordance with the long period condition of the uniformity in the valuation ratios. If (A.11) holds, (A.12) too holds in equilibrium. One can easily check that for the economy as a whole the following familiar national accounting relationship holds:

$$\sum J_i + G = \tilde{S}_f + \tilde{S}_h + (T - \rho B_{-1}) \qquad \text{(A.13)}$$

where $\tilde{S}_f = [1 + s_f(1 - t_r)r]\mathbf{xAp}$ = after-tax aggregate gross corporate saving; that is, for the economy as a whole, the sum of gross investment and government expenditure is equal to the sum of after-tax private sector saving (which in turn consists of after-tax gross corporate saving and after-tax household saving) and 'net' tax revenue (which is obtained by subtracting the payment of interest on T-bonds from the tax revenue collected by the government).

The equation system for an EDC economy with government expenditure consists of equations (A.1) to (A.11) , except (A.3) and (A.6) (plus an equation for the normalisation of prices): there are $5n+5$ unknowns (p_i, r, x_i, g_i, u_j, ϕ_i, h, G, T and ρ) in the same number of independent equations.

In analogy with equation (13), the ratio that the EDC level of output bears to its GAG counterpart (without government expenditure) is

$$\frac{\tilde{x}_i}{x_i^*} = \left(\frac{1-\tilde{\phi}_i}{1-\phi^*}\right)\left(\frac{1+s_f r-\phi^*}{1+s_f(1-t_r)r-\tilde{\phi}_i}\right)\left(\frac{\mathbf{A}_i\mathbf{p}}{\mathbf{A}_i\tilde{\mathbf{p}}}\right)z_i \equiv \tilde{\Phi}_i z_i \qquad \text{(A.14)}$$

where $\tilde{x}_i$, $\tilde{\phi}_i$ and $\tilde{\mathbf{p}}$ are the values determined in relation to government expenditure. (Note that, differently from equation (13), the ratio depends also on the prices of commodities; this is due to the introduction of tax rates in the price system.)

The impact of government expenditure on the quantity of output in the *i*-th industry can be expressed as an equivalent change in the amount of private investment, because, comparing $\tilde{\Phi}_i$ with its counterpart in the case of the absence of government expenditure (Φ_i) in equation (13), one gets from equation (A.14)

$$\frac{\tilde{x}_i}{x_i^*} = \Phi_i\left(\frac{\tilde{\Phi}_i}{\Phi_i}\right)z_i = \Phi_i\tilde{z}_i \qquad \text{(A.15)}$$

Thus, the introduction of government expenditure can be represented by the movement of PED from the position corresponding to the case where there was no government expenditure. Whether $\tilde{z}_i > z_i$ and also whether $\tilde{z}_i$ moves in step with G depend on how ϕ_i's are adjusted, which is in turn affected by how the government allocates G among the commodities. But it is natural to expect that PED should move with increases in G in the direction except South-West.