On Accounting ‘Capital’ in Horizontal Innovation Models

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1. Introduction

‘zero tolerance for intellectual sloppiness’

(Paul Romer)

1. Introduction

The *first* instance of ‘intellectual sloppiness’ of Romer (1990) [Park, 2007]:

- ‘the engine of economic growth
  = the increasing variety of intermediate goods’

- **but there is no *internal* criterion whatsoever by which to differentiate intermediate goods, except for the form of the Dixit-Stiglitz production function (which is an *external* criterion)**

1. Introduction

The *second* instance of ‘intellectual sloppiness’ of Romer (1990):

- use of both durable input(s) and nondurable input(s) in the setting of instantaneous production, where interest accrues to the former whilst not to the latter over the unit period of production (= an instant of time)

→ **logically inconsistent**
→ **the instantaneous period of production (with the zero length) should be replaced with the unit period of production of positive length**

1. Introduction

- in a HIM, at least one input (= design) ***is*** a durable (ever-lasting) input and at least one material input ***must*** be a nondurable input ('circulating capital')

* if **all** material inputs are ever-lasting
→ incompatible with the Dixit-Stiglitz specification (see Appendix)

1. Introduction

The *third* instance of 'intellectual sloppiness' of Romer (1990) [Park, 2010]:

- the unit period of production of positive length

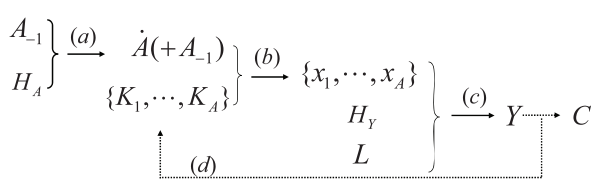
→ the measure of capital must reflect the effect of time (= interest)

→ the 'accounting measure of capital' is not the proper measure of capital on which the rate of interest is calculated

2. The 'accounting measure of capital'

2.1 The Romer (1990) model

The 'technical' structure of the model:



$(a)\quad \dot{A} = \delta H_A A_{-1}$

$(b)\quad x_i = \eta^{-1} K_i$

$(c)\quad Y = H_Y^{\alpha} L^{\beta} \sum_{i=1}^{A} x_i^{1-\alpha-\beta}$

$(d)\quad K_i = \eta x_i$

2. The 'accounting measure of capital'

2.1 The Romer (1990) model

The Y sector:

- technique: $Y = H_Y^{\alpha} L^{\beta} \sum x_i^{1-\alpha-\beta}$
- profits: $\pi_Y = Y - w_H H_A - w_L L - \sum \zeta_i x_i$
- max profits:

$\partial \pi_Y / \partial H_Y = 0; \quad \partial \pi_Y / \partial L = 0$

$\partial \pi_Y / \partial x_i = 0 \quad$ (→ demand for x_i : $\zeta_i = \zeta_i(x_i)$)

2. The 'accounting measure of capital'

2.1 The Romer (1990) model

The I sector:

- technique: η units of $Y \rightarrow 1$ unit of ith I
- monopolistic profits: $\pi_i = \zeta_i x_i - r\eta x_i$
- max profits: $d\pi_i / dx_i = 0$ (→ supply of x_i)
- monopolistic competition equilibrium:

$$P_A = \int_0^\infty e^{-rt}\pi_i dt \rightarrow rP_A = \pi_i$$

2. The 'accounting measure of capital'

2.1 The Romer (1990) model

The story of the I sector, told by Romer:

- the ith firm invests in developing the ith design
- the firm has a monopolistic right over the design
- the firm puts in ηx_i of Y to produce x_i of I
- the firm **rents** x_i to the Y sector for an infinite period, thereby obtaining a **rental** $\zeta_i x_i$ **every year**
- every year the firm maximises **monopolistic profits** (= annual rental – ***annual*** **variable cost** = $\zeta_i x_i - r\eta x_i$)

2. The 'accounting measure of capital'

2.1 The Romer (1990) model

The D sector:

- technique: $\dot{A} = \delta H_A A$
- profits: $\pi_A = P_A \dot{A} - w_H H_A$
- max profits: $d\pi_A / dH_A = 0 \rightarrow P_A \delta A = w_H$

2. The 'accounting measure of capital'

2.2 The measure of capital

foregone output:

$$\dot{K}(t) = Y(t) - C(t)$$

the "**accounting measure of capital**"
= accumulated foregone output:

$$K(t) = \int_0^t \dot{K}(v)dv = \sum_{i=1}^{A(t)} K_i = \sum_{i=1}^{A(t)} \eta x_i = \eta \sum_{i=1}^{A(t)} x_i$$

(symmetry)

$$K(t) = \eta x A(t)$$

2. The 'accounting measure of capital'

2.2 The measure of capital

The role of the 'accounting measure of capital':

$$\max_C \int_{t=0}^{\infty} e^{-\rho t}\left(\frac{C(t)^{1-\sigma}-1}{1-\sigma}\right)dt$$

s.t.

$$Y = C + \dot{K}$$
$$Y = w_H H_Y + w_L L + \zeta x A$$
$$= w_H H_Y + w_L L + r P_A A + rK$$
$$P_A \dot{A} = w_H H_A$$

- the Euler equation: $\frac{\dot{C}}{C} = \frac{r-\rho}{\sigma}$

2. The 'accounting measure of capital'

2.2 The measure of capital

comparison with the Solow model:

- foregone output: $\dot{K}(t) = Y(t) - C(t)$
- capital as accumulated foregone output:

$$K(t) = \int_0^t \dot{K}(v)dv$$

- $\max_C \int_{t=0}^{\infty} e^{-\rho t}\left(\frac{C(t)^{1-\sigma}-1}{1-\sigma}\right)dt \quad s.t. \quad C + \dot{K} = wL + rK$
- the Euler equation: $\frac{\dot{C}}{C} = \frac{r-\rho}{\sigma}$

2. The 'accounting measure of capital'

2.2 The measure of capital

The measure of 'capital' on which the rate of interest is calculated is a **quantity which does *not involve the rate of interest***.

- Romer: **'accounting measure of capital'**
- Barro and Sala-i-Martin: **'assets'**
- Jones: **'capital'** (2002), **'financial assets'** (2005)

2. The 'accounting measure of capital'

2.2 The measure of capital

Questions:

- How can a HIM, which is a 3-sector model, have the same 'structure' as the Solow model, which is a 1-sector model?
- Is r calculated in reference to K in a HIM a correct measure of the (uniform) rate of interest of the economy?

3. Value and capital

3.1 The 'value' structure of the Romer model

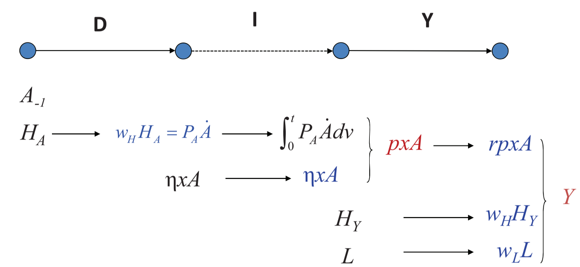
An alternative, equivalent, story of the I sector:

- the *i*th firm invests in developing the *i*th design
- the firm has a monopolistic right over the design
- the firm puts in ηx_i of Y to produce x_i of I
- the firm **sells** x_i to the Y sector at the price of p_i
- the firm sets p_i that maximises **profits**

(= total revenue – total cost $= p_i x_i - (\eta x_i + P_A)$), taking P_A as given (already determined in the D sector)

3. Value and capital

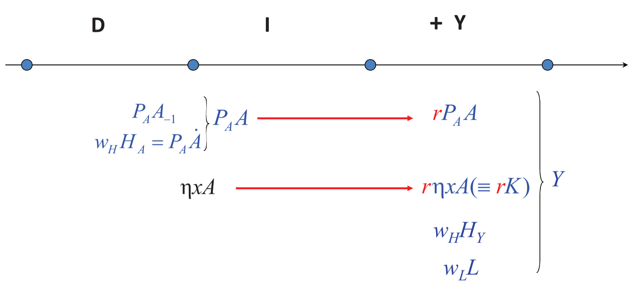
3.1 The 'value' structure of the Romer model



3. Value and capital

3.1 The 'value' structure of the Romer model

… effectively equivalent to:



3. Value and capital

3.1 The 'value' structure of the Romer model

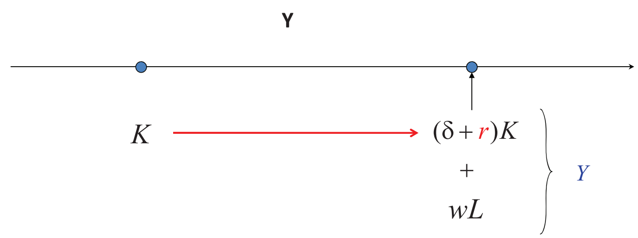
… interpreted this way:

- there are firms producing new designs using the stock of old designs and human capital: $\dot{A} = \delta H_A A$
- profit max of designs determines their price: $w_H H_A = P_A$
- firms producing Y use, as inputs, the stock of designs, foregone Y goods ('capital', $\sum \eta x_i$), human capital (H_Y) and physical labour (L)
- designs (and human capital H_A associated with them) and 'capital' are durable inputs → **interest cost at *r***
- human capital H_Y associated with Y and physical labour are paid ex post → **no interest cost**

3. Value and capital

3.1 The 'value' structure of the Romer model

comparison with the Solow model:



3. Value and capital

3.1 The 'value' structure of the Romer model

the price equation: $px = P_A + \eta x$

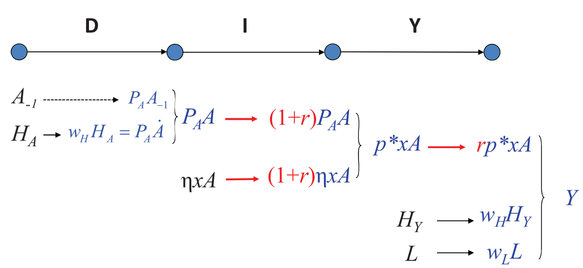
the rental: $\zeta x = rpx = r(P_A + \eta x)$

the value of intermediate goods ('financial asset'): $V = pxA = P_A A + K$

interest: $Z = rV = rP_A A + rK$

3. Value and capital

3.2 Value and capital, properly set



3. Value and capital

3.2 Value and capital, properly set

the price equation: $p^*x = (1+r)(P_A + \eta x) = (1+r)px$

the rental: $\zeta^* x = rp^* x = r(1+r)(P_A + \eta x)$

the value of intermediate goods: $V^* = p^* xA = (1+r)P_A A + (1+r)K$

interest: $Z^* = rV^* = r(1+r)V$

3. Value and capital

3.2 Value and capital, properly set

the constraint:

$$C+\dot{K}=w_H H_Y + w_L L + r(1+r)P_A A + r(1+r)K$$

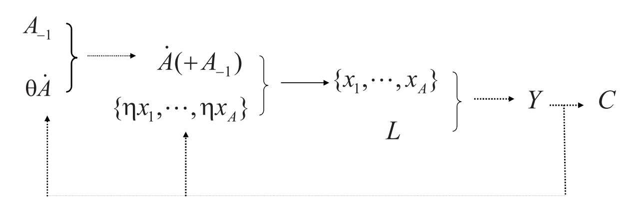
$$P_A\dot{A}=w_H H_A$$

the Euler equation: $\frac{\dot{C}}{C}=\frac{r(1+r)-\rho}{\sigma}$

(3. Value and capital)

(3.3 The Barro and Sala-i-Martin model)

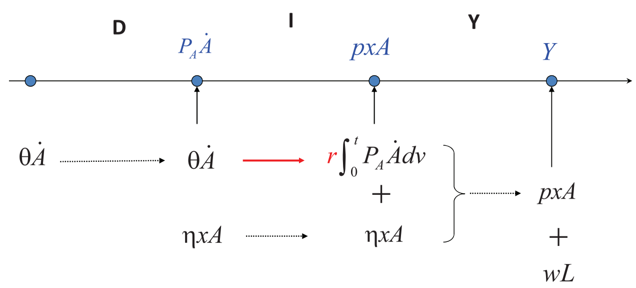
The 'technical' structure of the model:



(3. Value and capital)

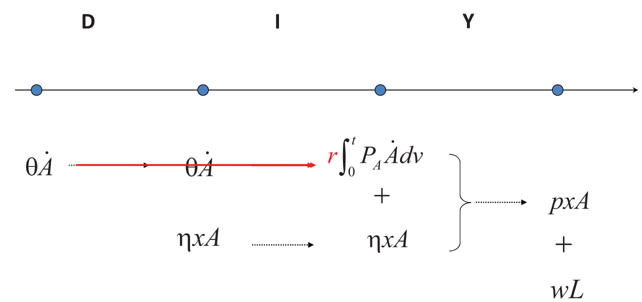
(3.3 The Barro and Sala-i-Martin model)

The 'value' structure of the model:



(3. Value and capital)

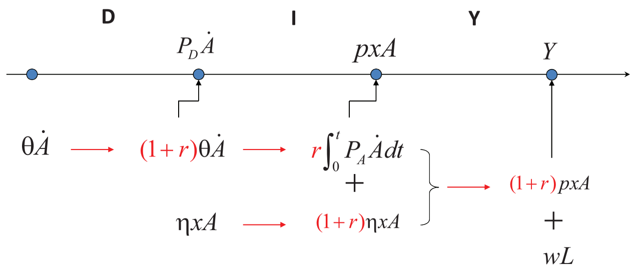
(3.3 The Barro and Sala-i-Martin model)



(3. Value and capital)

(3.3 The Barro and Sala-i-Martin model)

The 'value' structure of the model, with time considered:



4. Conclusion: the problem of value

The problem of value is *inevitable* when

(i) there are a multiple number of goods

→ the standard of measurement

(ii) production takes time

→ interest as the price for waiting

→ the compounding of interest as time passes by

4. Conclusion: the problem of value

The Romer model avoids the problem of value by way of an incorrect measure of a quantity on which to calculate the rate of interest.

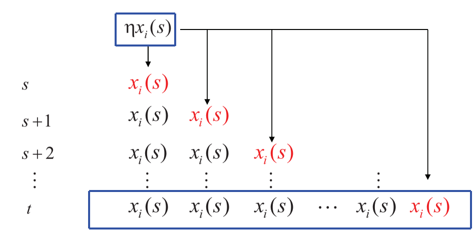
Other horizontal innovation models also avoid the problem of value in various ways, which are all reduced to

assuming away time in production

(or, inconsistently applying time to different inputs)

Appendix: the case of *all* durable inputs

A.1 Infinite production with finite input



Appendix: the case of *all* durable inputs

A.1 Infinite production with finite input

at period t, the stock of the ith intermediate good which was first produced at period s is

$$Q_i(s,t) \equiv (t-s+1)x_i(s) \quad > \; x_i(s)$$

Appendix: the case of *all* durable inputs

A.1 Infinite production with finite input

problem 1: the production of the Y good at period t, using the stock of the (durable) I goods of (4):

$$Y(t) = H_Y^{\alpha} L^{\beta} \sum_{i=1}^{A(t)} \left[(t-s+1)x_i(s)\right]^{1-\alpha-\beta}$$

in contrast, Romer's production function:

$$Y(t) = H_Y^{\alpha} L^{\beta} \sum_{i=1}^{A(t)} x_i^{1-\alpha-\beta}$$

Appendix: the case of *all* durable inputs

A.1 Infinite production with finite input

problem 2: the quantity, measured in the Y good, of the total stock of the intermediate goods at period t :

$$\sum_{s=1}^{t} \sum_{i=1}^{\dot{A}(s)} (t-s+1)\left[\eta x_i(s)\right]$$

$$> \; \sum_{s=1}^{t} \sum_{i=1}^{\dot{A}(s)} \eta x_i(s) \;\; \equiv \; K(t)$$

→ **a given quantity ηx_i of the Y good is associated with an ever-increasing quantity of the same Y good, $(t-s+1)\eta x_i$!**

Appendix: the case of *all* durable inputs

A.2 Solutions

To solve this problem,

either intermediate goods

or the final good (as capital good)*

or both**

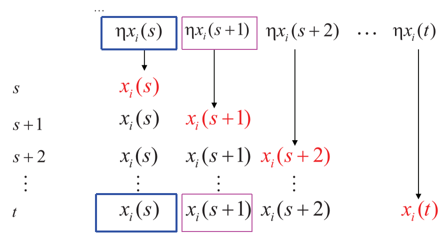
must be nondurable.

* Romer (1990)
** Barro and Sala-i-Martin (2004, Ch. 6)

Appendix: the case of *all* durable inputs

A.2 Solutions

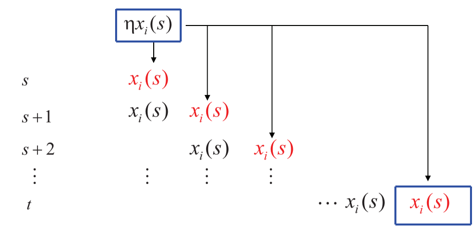
(1) the case of non-durable Y good and durable I goods:



Appendix: the case of *all* durable inputs

A.2 Solutions

(2) the case of durable Y good and non-durable I goods:



Appendix: the case of *all* durable inputs

A.2 Solutions

(3) the case of non-durable Y good and non-durable I goods: