THE ROLE OF THE UTILITY FUNCTION IN THE ESTIMATION OF PREFERENCE PARAMETERS

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ABSTRACT

Within the modern mainstream theory of consumption, the main characteristics of individual preferences are represented by a definite set of parameters: risk aversion, prudence, and the elasticity of intertemporal substitution. To give quantitative definition to each of these parameters is crucial, but the results of the empirical literature are controversial. Among the reasons for this failure, a possible, neglected role may have been played by the need to specify a utility function to perform the estimation. This paper presents a number of simulation exercises, which show that the same saving behaviour can be associated with quite different values of the preference parameters depending on the utility function adopted. The analysis suggests that the research devoted to estimating preference parameters has been affected by the constraints imposed, on the one hand, by the quantitative definition of the parameters, and, on the other, by the utility functions adopted.

KEYWORDS: Consumption; Elasticity of Intertemporal Substitution; Risk Aversion; Precautionary Saving; Utility function.

JEL CODES: D120; D150; D810

1. INTRODUCTION

Within the intertemporal utility maximization model that is at the basis of the modern mainstream view of consumer behaviour, the main characteristics of individual preferences are represented by a definite set of parameters. Such parameters synthetically describe the crucial features of the observed consumption behaviour, while at the same time representing the properties of individual preferences from which such behaviour derives. Such features are: i) the tendency of agents to prefer a certain amount of consumption over an aleatory amount with the same expected value, that is, consumers are risk averse; ii) the tendency of agents to save as a result of the presence of uncertainty, i.e., consumers are prudent; and iii) the tendency of agents to save as a result of the incentive represented by interest on saved sums, i.e., consumers have an intertemporal motive to save. A specific parameter corresponds to each of these crucial properties of preferences: a coefficient describing (absolute or relative) risk aversion, a coefficient describing (absolute or relative) prudence, and an elasticity of intertemporal substitution.

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A substantial strand of research has attempted to give adequate quantitative definition to each of these parameters, and to specify the intensity of each of them through appropriate numerical estimates. Such empirical literature originates within the approach to consumption theory based on the Euler equation, which predicts that consumers allocate resources over time in order to maximize lifetime utility derived from consumption subject to an intertemporal budget constraint. The first-order conditions of the optimization problem give rise to the well-known Euler equation:

$$u'(C_t) = (1 + \delta)^{-1} \mathbb{E}_t[(1 + r_t) \cdot u'(C_{t+1})]$$

where \(u(\cdot)\) is the within-period utility function, \(C_t\) is consumption at time \(t\), \(\delta\) is the rate of time preference, \(r_t\) is the (stochastic) interest rate between \(t\) and \(t+1\), and \(\mathbb{E}\) is the expectations operator.¹

The idea that originated the empirical literature on preference parameters, proposed by Hall (1978), consists in deriving a set of orthogonality conditions from equation (1), which can be verified empirically and allows for both testing the validity of the model and estimating the structural parameters of the utility function. However, the results of this literature are controversial, because the estimates are characterized by high variability and there appears to be little consensus regarding the magnitude of the parameters. In fact, while some scholars consider the Euler equation an effective and convenient tool for estimating preference parameters, others seem rather sceptical. For example, Attanasio and Weber (2010, p. 741) claim that «Euler equations are remarkably useful because they let researchers estimate important preference parameters in a relatively robust way». Conversely, Carroll (2001, p. 1) notices that «[u]nfortunately, despite scores of careful empirical studies using household data, Euler equation estimation has not fulfilled its early promise to reliably uncover preference parameters».

For that matter, the evolution of economic theory in recent decades has made it more and more important to determine the magnitude of preference parameters. First, the model of intertemporal utility maximization would prove devoid of definite theoretical content without a specification of individual preferences. In fact, to state that the choice between consumption and saving derives from the solution of a problem of intertemporal maximization, without any further specification, would only constitute a very general framework of thought. The capability of such framework to provide indications about saving behaviour depends on the specification of the preferences of the consumer. Second, in response to the implications of the Lucas critique, mainstream macroeconomics has given up on the

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¹ Equation (1) is obtained when assuming that the agent maximizes expected utility over a single consumption good, forms rational expectations about future uncertain incomes and interest rates, and discounts future utility at a positive and constant rate; utility is additively separable across periods and a single financial asset can be used to move resources over time, with uniform lending and borrowing rates and no credit rationing. The assumptions on which standard optimizing models of intertemporal allocation of consumption rely are not innocuous. Objections to those assumptions include the axioms of expected utility, exponential discounting, and time additivity, to cite just some examples. However, discussing this set of assumptions goes beyond the purposes of this work.
idea that it is possible to identify an aggregate consumption function and relies instead on an entirely microfounded description of consumption. As a result, the general equilibrium models that underpin modern macroeconomic literature necessarily require the specification of the utility function of the representative agent, including the specification of the structural parameters that characterize it. Such parameters either have to be estimated within the general equilibrium model itself or have to be inferred from the empirical microeconometric literature. The analysis that such general equilibrium models offer of macroeconomic phenomena and their conclusions depend in no small part on the value attributed to the structural parameters, including those representing preferences. Third, specification of the utility function and estimation of its parameters are crucial for policy: since the parameters are a basic feature of both the microeconomic model of intertemporal utility maximization and the general equilibrium models, they are crucial for devising and implementing policy measures. For example, within a DSGE model, a different value attributed to the elasticity of intertemporal substitution (EIS) implies a different reaction to a monetary policy shock. To mention just one other reason for accurate estimates of the EIS to be important, the public finance literature relies on this parameter to evaluate the welfare effects of capital income taxation, which indeed requires quantifying the contribution of interest rates in explaining consumption growth.

The crucial role of the estimation of preference parameters for policy purposes may further be appreciated if we reflect on the role of prudence in evaluating the benefits of a programme of social security. Once the presence of precautionary saving is admitted, it must in fact be concluded that uncertainty over future consumption generates, at least for some households, a welfare loss, which may only be quantified once the parameter representing prudence is given. The magnitude of the parameter is therefore essential to determining whether the introduction of a social welfare scheme aimed at reducing the impact of uncertainty on future incomes would bring about an increase in welfare.

The purpose of this paper is to provide a critical assessment of the results of the empirical literature devoted to the estimation of the parameters characterizing utility functions. In particular, our analysis suggests that the functional form adopted in the estimation of the parameters might be crucial for the estimates obtained. In order to highlight the im-

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2 This is effectively shown by Havránek et al. (2015), who replicate the well-known model of Smets and Wouters (2007) by attributing different values to the EIS. The experiment shows that the dynamics of consumption and investment changes dramatically if the parameter varies between 1.5, as in Smets and Wouters, and 0.1, a value consistent with a large part of the empirical evidence.

3 See Summers (1981) for an early contribution. Attanasio and Wakefield (2010) simulate a life cycle model to analyse the determinants of the effect on savings of changes in the interest rate due to changes in asset returns taxation. In their model, when the interest rate increases from 2% to 2.5%, this implies an average increase in wealth at retirement of 18% if the EIS is set at 1, which reduces to 4% when the EIS is set at 0.25 (see Section 7.2.1).

4 Browning and Lusardi (1996) offer both a discussion of this case and a numerical example. In their two-period model, a consumer with a coefficient of relative prudence set to 2 would be just as well off if he had a certain income 8% lower than the expected value of his income (see pp. 1801–1804). This numerical example will be further discussed in Section 5.2.
portance of the functional form adopted, a number of simulation exercises are conducted, extending a numerical example proposed by Browning and Lusardi (1996) to the case of different utility functions and slightly more complicated hypotheses about the stochastic framework faced by the consumer. This simple simulation exercise is meant to show that the same saving behaviour can be associated with quite different attitudes towards time and uncertainty – that is, to different magnitudes of the parameters – depending on the utility function adopted. Moreover, the simulation exercise is associated with a more formal and general analysis of the impact of parameters’ values on the saving behaviour of the consumer, conducted by means of a decomposition of the rate of growth of consumption derived by the Euler equation.

The paper is organized as follows. In Section 2, after discussing the meaning of each preference parameter, we stress the links existing between them. We thus show that while measuring different aspects of preferences, the main parameters, because of the way they are defined, have close mathematical relationships between each other, which impose strict constraints on the estimates of the parameters themselves. In Section 3, we provide a survey of the most popular utility functions adopted in the literature, emphasizing their main properties in terms of preference parameters. Section 4 briefly reviews the empirical evidence on the parameters collected since Hall’s contribution, with a focus on the choice of the utility function adopted for the estimation. Section 5 is dedicated to the discussion of the simulation exercises. Section 6 concludes.

2. Preference Parameters

2.1. Summarizing the attitude toward risk: the coefficients of risk aversion

An agent is risk averse if, given the choice between the same amount of consumption achievable through a gamble or through certainty, he will opt for certainty. The risk attitude is related to the curvature of the utility function: if the utility function is concave, then the utility of the expected value of an uncertain amount is greater than the expected utility of that amount.

The modern literature on risk aversion began with the work of Pratt (1964) and Arrow (1965), who independently proposed formal measures of risk aversion. Since the degree of risk aversion depends on the curvature of the utility function, they suggested that it could be measured by means of the second derivative. To ensure risk attitudes are preserved by increasing transformations of the utility function, the second derivative needs to be normalized. Hence, the Arrow-Pratt measures of absolute and relative risk aversion are, respectively:

\[ A(C_t) = -\frac{u''(C_t)}{u'(C_t)} \quad \text{and} \quad a(C_t) = -\frac{u''(C_t) \cdot C_t}{u'(C_t)}. \]

The index of absolute risk aversion is suited to the comparison of attitudes towards additive risks, whose outcomes are absolute gains or losses from current consumption, where-
as the measure of relative risk aversion is useful to evaluate multiplicative risks, whose outcomes are percentage gains or losses from current consumption.

Both Pratt and Arrow in their contributions suggested absolute risk aversion to be decreasing and the subsequent literature has indeed widely agreed that preferences, in order to be plausible, should exhibit decreasing, or at most constant, absolute risk aversion. More controversial appears to be the behaviour of relative risk aversion: it is usually believed that wealthier people are willing to bear more risk than poorer people, which is why a decreasing absolute risk aversion is considered plausible, but the hypothesis of decreasing relative risk aversion is much stronger, because it implies that agents become less risk averse with regard to gambles that are proportional to their wealth as their wealth increases. In addition, a risk averse agent with decreasing relative risk aversion will exhibit decreasing absolute risk aversion, but the converse is not necessarily the case. Generally, assuming constant relative risk aversion is thought to be a quite plausible assumption.

2.2. Measuring the precautionary motive for saving: relative and absolute prudence

When preferences exhibit prudence and the consumer faces uninsurable income uncertainty, he is willing to protect himself against the fall in consumption that would result from a fall in income by reducing current consumption: prudence leads agents to treat future uncertain income cautiously and not to spend as much currently as they would if future incomes were certain. The saving that results from the knowledge that the future is uncertain is known as precautionary saving.

Leland (1968) demonstrated that precautionary saving requires convex marginal utility in addition to risk aversion. In fact, an increase in uncertainty about next period consumption, if marginal utility is convex, will increase future marginal utility, so that current consumption will have to decrease in order to bring the current marginal utility back into equality as prescribed by equation (1).

The idea of a precautionary motive for saving is very old, but received formal treatment only in the late 1960s, with the work of Leland (1968), Sandmo (1970), Mirman (1971), and Drèze and Modigliani (1972). However, it was not until later, within the framework delineated by the Euler equation approach to consumption, that a specific preference parameter was proposed as representing the effects of uncertainty on saving behaviour. In parallel to the Arrow–Pratt coefficients, Kimball (1990) introduced two measures of the intensity of the precautionary saving motive which rely on the convexity of marginal utility: a measure of absolute prudence and a measure of relative prudence,

\[ P(C_t) = -\frac{u''''(C_t)}{u''(C_t)} \quad \text{and} \quad p(C_t) = -\frac{u''''(C_t) \cdot C_t}{u''(C_t)}, \]

which summarize the size of the reaction to an increase in uncertainty.

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\[ ^5 \text{Since } a(C) = A(C) \cdot C, a'(C) < 0 \text{ implies } A'(C) < 0. \]
Like risk aversion, prudence can be increasing or decreasing (or constant) in consumption: the additional savings induced by an increase in uncertainty will be smaller the richer the consumer when prudence is decreasing, and vice versa. In accordance with Kimball’s suggestion, it is usually believed that a precautionary saving motive which decreases in intensity with wealth is more plausible.

2.3. The aversion to temporal fluctuations of consumption: the EIS

According to the model of intertemporal utility maximization, the consumption profile of the agent should be designed so as to take advantage of the interest rate: if the interest rate between \( t \) and \( t + 1 \) is high, then there is an additional incentive to postpone consumption in \( t \) in favour of \( t + 1 \). This is immediately apparent when we consider the Euler equation in the case of a constant interest rate:

\[
\frac{u'(C_t)}{1+\delta} = \frac{1+r_t}{1+\delta} \mathbb{E}_t[u'(C_{t+1})]
\]

The equilibrium condition predicts that consumption increases over time if \( r \) exceeds \( \delta \) and falls if \( r \) is less than \( \delta \).

The magnitude of the effect on savings of changes in the interest rate depends on the elasticity of intertemporal substitution, which represents the proportional change in consumption growth that must follow to a one percent change in the interest factor:

\[
EIS(C_t, C_{t+1}) = \frac{d(C_{t+1}/C_t)}{d(1+r_t)} \frac{1}{1+r_t}
\]

The concept of intertemporal elasticity of substitution has been defined by Browning (1985/2005), Mankiw et al. (1985), and Hall (1988). Following Browning, the EIS is usually defined as the derivative of log consumption with respect to the log of the interest rate and measured by the reciprocal of the consumption elasticity of marginal utility:

\[
EIS(C_t) = -\frac{u'(C_t)}{C_t \cdot u''(C_t)}
\]

The interest factor represents the price of present consumption relative to future consumption, so when it increases current consumption decreases because of the substitution effect. At the same time, with a higher interest rate, any target level of future consumption is achieved with less savings, which implies an income effect going in the opposite direction of the substitution effect. The income effect becomes more important relative to the substitution effect for lower levels of the EIS.

A higher interest rate also implies higher discount factors to be applied to future incomes, thereby inducing a wealth effect, which reinforces the substitution effect leading to a decrease in current consumption. Since the effect of changes in the interest rate on current consumption depends on the interplay of income, substitution, and wealth effects, it is

\[\text{The presence of a wealth effect, in addition to the traditional substitution and income effects, is analysed by Summers (1981).}\]
ambiguous; but its effect on the growth rate of consumption can never be negative because that would imply non-concave utility. Therefore, the elasticity of intertemporal substitution must be positive or, at most, null.

The EIS depends on the curvature of the utility function: if the marginal utility of consumption decreases very quickly as consumption rises, the agent does not need to change his consumption much to take advantage of intertemporal incentives, since the profile of marginal utility can be matched to the changed interest rate with little change in consumption. In fact, the EIS measures the aversion to intertemporal fluctuations of consumption: a lower value of the EIS implies that the agent wants to achieve a smoother path of consumption across the life cycle, and is therefore less willing to change consumption in response to a change in the interest rate.

2.4. The links between preference parameters

The degree of risk aversion, the intensity of the precautionary saving motive and the degree of intertemporal substitutability, describe different aspects of individual preferences: the measures of risk aversion indicate how much the agent is willing to sacrifice his consumption in the best case scenario in order to achieve a higher level of consumption in the worse; the magnitude of prudence measures how an increase in uncertainty about future income will affect current consumption; the elasticity of intertemporal substitution indicates the extent to which the agent is willing to substitute future consumption for current consumption in response to the incentive given by interest rates.

However, close relationships exist between the different preference parameters because of their analytical definition. The derivation of most of those links is basic, and in fact they were highlighted long before the appearance of the empirical literature devoted to the estimation of parameters. Nevertheless, they still often appear to be ignored.

At the very beginning of the formal analysis of precautionary saving, Leland (1968), investigating the conditions required for uncertainty to stimulate savings, demonstrated that for the utility function to exhibit prudence it is necessary that the coefficient of absolute risk aversion be decreasing. Mirman (1971) and Drèze and Modigliani (1972) further analysed the links between precautionary saving and the concavity of the utility function. It was then clear that the necessary and sufficient condition for absolute risk aversion to be decreasing is that the function \(-u'(\cdot)\) be more concave than the function \(u(\cdot)\), that is, that absolute prudence be larger than absolute risk aversion. This means that, first of all, for absolute risk aversion to decrease in consumption, so as to meet the Arrow and Pratt hypothesis, a necessary condition is that the utility function have a positive third derivative. Secondly, when a decreasing absolute risk aversion (DARA) utility function is chosen, it is automatically implied that the consumer is more inclined to take precautions than to avoid risk.
More generally, the relationship between absolute prudence and absolute risk aversion can be seen by differentiating $A(C_t)$ to get

$$P(C_t) = A(C_t) - \frac{A'(C_t)}{A(C_t)} \quad (3)$$

and, multiplying both sides in (3) by the consumption level, relative prudence appears as the difference between relative risk aversion and the consumption elasticity of absolute risk aversion:

$$p(C_t) = a(C_t) - \frac{C_t \cdot A'(C_t)}{A(C_t)} \quad (4)$$

From (3) it is easy to see that if the utility function exhibits DARA, then $A'(C_t) < 0$ and the consumer is more prudent than risk averse. This happens with isoelastic preferences. In contrast, under constant absolute risk aversion (CARA), $A'(C_t) = 0$ and risk aversion and prudence are exactly the same, as in the exponential utility case. And, finally, with increasing absolute risk aversion (IARA), $A'(C_t) > 0$ and the consumer is more risk averse than prudent, which occurs under quadratic preferences. Thus, the magnitude of relative prudence is closely related to the magnitude of relative risk aversion, and yet it has important behavioural implications of its own. In fact, a value $p(C_t) \leq 0$ indicates a lack of precautionary motives, and $p(C_t) > 0$ implies a tendency to undertake precautionary saving in the face of additive risk, while $p(C_t) > 2$ implies precautionary saving under multiplicative risk (Eisenhauer, 2000).

The behaviour of prudence as consumption increases is related to risk aversion as well: Kimball (1993) shows that assuming decreasing absolute prudence is a stronger hypothesis than assuming decreasing absolute risk aversion, since the former implies the latter.\(^7\) Such a clear-cut link between the magnitude and behaviour of these parameters has long been generally ignored in the literature on the consumption Euler equation and still appears to be often neglected. For example, Blanchard and Mankiw (1988), after discussing the case of CARA, in which prudence and risk aversion are the same, argue that «In general, there need not be any tight relation between the coefficients of risk aversion and the coefficients of prudence» (p. 174). Likewise, Browning and Lusardi (1996), with reference to CRRA utility, claim that «In general there is no necessary link between risk aversion and prudence» (p. 1806).

However, the link between risk aversion and intertemporal substitution is even closer. The standard version of the intertemporal maximization model, indeed, implies a double additivity induced by the simultaneous assumptions of intertemporal separability and expected utility (additivity over periods and over states of nature), which entails a negative relation between risk aversion and intertemporal substitution: consumers who are risk averse will be unresponsive to intertemporal incentives, while those who are willing to re-

\(^7\) For a discussion of the relationship between the monotonicity of absolute risk aversion and absolute prudence, see Maggi et al. (2006).
allocate their consumption in response to changes in the interest rate will display relatively little aversion towards risk. That happens because both parameters are governed by the concavity of the utility function:

\[ \text{EIS}(C_t) = -\frac{u'(C_t)}{C_t u''(C_t)} = \frac{1}{a(C_t)} \]

There are authors who do not feel uncomfortable in accepting the link between the two parameters, such as Deaton, according to whom,

\[ \text{time and uncertainty are so intimately connected that (at least to this writer) there is strong intuitive support for a relationship between attitudes towards risk and attitudes towards substitution.} \]

In an uncertain world, the substitution of future consumption for current consumption inevitably increases exposure to risk, and those who are willing to contemplate the former must be willing to face the latter (Deaton, 1992, p. 20).

By contrast, many other scholars believe that risk aversion and intertemporal substitution are independent aspects of consumer preferences, so that any formulation that mixes them up is incorrect. Hall is one of them:

\[ \text{[I]t would be desirable to eliminate this automatic connection between intertemporal substitution and risk aversion (Hall, 1988, p. 343).} \]

Of the same opinion is Weil, who claims that

\[ \text{[a]version to risk and aversion to intertemporal substitution are two conceptually distinct attributes, and should be parameterized independently of each other (Weil, 1990, p. 29).} \]

More recently, Kimball and Weil argue that

\[ \text{[b]ecause it does not distinguish between aversion to risk and aversion to intertemporal substitution, the traditional theory of precautionary saving based on intertemporal expected utility maximization is a framework within which one cannot ask questions that are fundamental to the understanding of consumption in the face of labor income risk (Kimball and Weil, 2009, p. 245).} \]

Although it may be the case that a negative correlation exists between risk aversion and intertemporal substitution, constraining the response to the incentive given by interest rates to be, \textit{a priori}, exactly the reciprocal of the coefficient summarizing the attitude towards risk appears to be quite restrictive. Besides, as Campbell points out,

\[ \text{[r]isk aversion describes the consumer’s reluctance to substitute consumption across states of the world and is meaningful even in an atemporal setting, whereas the elasticity of intertemporal substitution describes the consumer’s willingness to substitute consumption over time and is meaningful even in a deterministic setting (Campbell, 2003, p. 828).} \]

\[ \text{See also Selden (1978), Barsky (1989), and Campbell (1993) as contributions arguing that individual attitudes towards risk and intertemporal fluctuations are unrelated.} \]
In order to disentangle the two parameters, one needs to give up one of the two sources of additivity, either rejecting the axioms of expected utility theory or abandoning intertemporal separability. The former route was pursued by Epstein and Zin (1989) and Weil (1990), who developed the nonexpected utility model of choice proposed by Kreps and Porteus (1978). The resulting functional form, often called Epstein–Zin–Weil (EZW) recursive utility, has been increasingly adopted in the Euler equation literature on consumption in recent years. In particular, because of the flexibility which it seems to provide in terms of the magnitude of preference parameters, EZW utility has been employed in the empirical work aiming at estimating risk attitudes and the degree of intertemporal substitutability. This functional form is illustrated in the next section, where the main preference specifications adopted in the consumption literature are presented; at this stage, it is just worth stressing that, while keeping intertemporal substitution and risk aversion separated, EZW preferences imply a degree of prudence implicit in the definition of the other two parameters and, moreover, they require the introduction of an additional aspect of preferences: the so-called “timing premium”, which relates to the preference for the timing of resolution of uncertainty. Therefore, the higher degree of flexibility of EZW utility does not imply that the links between preference parameters are satisfactorily relaxed.

As for the alternative route toward a separation of the coefficient of relative risk aversion and the elasticity of intertemporal substitution, which retains expected utility while ruling out intertemporal separability of preferences, this has been undertaken through the introduction of habit formation in the utility function. Habit-forming preferences have played a significant role in the consumption literature within the Euler equation approach ever since the works of Deaton (1986), Abel (1990), and Constantinides (1990), and have been employed in works aiming at the estimation of preference parameters, such as Gayle and Khorunzhina (2017) and Khvostova et al. (2016). However, most studies relying on Euler equation estimation have found little or no evidence of habits, such as Meghir and Weber (1996) and Dynan (2000).

3. Utility Functions

This section presents the most popular within-period utility functions adopted in the literature on consumption, illustrating the Euler equation associated with each preference specification and the implications of the functions in terms of parameters’ values and behaviour. Most of them, namely quadratic, exponential, and isoelastic preferences, belong to the family of hyperbolic absolute risk aversion (HARA) functions, which imply that the inverse of absolute risk aversion (called absolute risk tolerance) is linear in consumption. Together with nonexpected utility Epstein–Zin–Weil preferences, which are also discussed, these functional forms are illustrated in Table 1. The quantitative definition of the parameters for each utility function is presented in Table 2.
### Table 1. Functional form and Euler equation of HARA and EZW preferences

<table>
<thead>
<tr>
<th>Expected utility HARA preferences</th>
<th>Marginal utility</th>
<th>Euler equation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Within-period utility function</strong></td>
<td><strong>Marginal utility</strong></td>
<td><strong>Euler equation</strong></td>
</tr>
<tr>
<td>Quadratic</td>
<td>$u(C_t) = aC_t - \frac{b}{2}C_t^2$</td>
<td>$u'(C_t) = a - bC_t$</td>
</tr>
<tr>
<td>Exponential</td>
<td>$u(C_t) = -\frac{e^{-\alpha C_t}}{\alpha}$</td>
<td>$u'(C_t) = e^{-\alpha C_t}$</td>
</tr>
<tr>
<td>Isoelastic</td>
<td>$u(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma}$</td>
<td>$u'(C_t) = C_t^{-\gamma}$</td>
</tr>
<tr>
<td>Logarithmic</td>
<td>$u(C_t) = \ln C_t$</td>
<td>$u'(C_t) = \frac{1}{C_t}$</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Nonexpected utility recursive EZW preferences</th>
<th>Marginal utility</th>
<th>Euler equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Utility function</td>
<td>Marginal utility</td>
<td>Euler equation</td>
</tr>
<tr>
<td>$V_t = \left{C_t^{1-\frac{1}{\sigma}} + \beta[\mathbb{E}<em>t(V</em>{t+1}^{1-\gamma})]^{1-\frac{1}{\sigma}}\right}^{\frac{1}{\sigma}}$</td>
<td>$\frac{\partial V_t}{\partial C_t} = \left(\frac{V_t}{C_t}\right)^{\frac{1}{\sigma}} - 1 = \beta\mathbb{E}<em>t\left[\left(\frac{C</em>{t+1}}{C_t}\right)^{1-\gamma}\right]^{\frac{1}{\sigma}}$</td>
<td></td>
</tr>
</tbody>
</table>

| Table 2. Preference parameters for HARA and EZW preferences |
|----------------------------------|-----------------|----------------|
| Quadratic preferences | Exponential preferences | Isoelastic preferences | Logarithmic preferences | Recursive EZW preferences |
| $A(C)$ | $\frac{b}{a - bC}$ | $\alpha$ | $\frac{1}{C}$ | $\frac{1}{C}$ | $\frac{1}{C}$ |
| $a(C)$ | $\frac{b}{a - bC}C$ | $aC$ | $\gamma$ | $1$ | $\gamma$ |
| $EIS(C)$ | $\frac{a - bC}{bC}$ | $\frac{1}{aC}$ | $\frac{1}{\gamma}$ | $1$ | $\sigma$ |
| $P(C)$ | $0$ | $\alpha$ | $\frac{1 + \gamma}{C}$ | $\frac{2}{C}$ | $\frac{\gamma(1 + \sigma)}{C}$ |
| $p(C)$ | $0$ | $aC$ | $1 + \gamma$ | $\frac{2}{C}$ | $\frac{\gamma(1 + \sigma)}{C}$ |

### 3.1. Quadratic preferences: certainty equivalence

Earlier attempts at testing the utility maximization model of consumption relied on the special case of quadratic preferences, known in the literature as the Permanent Income model with certainty equivalence or, simply, certainty-equivalence model (examples can be found in Hall, 1978; Flavin, 1981; Campbell, 1987).

Under the hypothesis of quadratic preferences, the Euler equation (2) rewrites as
\[
a - bC_t = \left(\frac{1 + r}{1 + \delta}\right)\mathbb{E}_t[(a - bC_{t+1})]
\]
and, assuming that the interest rate and the rate of time preference are equal, one gets the famous random walk result:

\[ a - bC_t = a - bE_t[C_{t+1}] \]

\[ E_t[C_{t+1}] = C_t. \tag{5} \]

Such a strong implication of quadratic preferences depends on the linearity of marginal utility: since the expected marginal utility of consumption is the same as the marginal utility of expected consumption, uncertainty about future consumption has no impact on current consumption.

The impossibility of capturing the precautionary motive for saving is the reason why the model with quadratic preferences is universally considered to be very misleading in the presence of uncertainty.\(^{10}\) However, the lack of prudence is not the only reason why quadratic utility is unappealing. In fact, it also implies increasing absolute risk aversion, which means that the amount of consumption that agents are willing to give up to avoid a given amount of uncertainty about the level of consumption rises as they become wealthier. This property is considered «unappealing on theoretical grounds and strongly counterfactual (riskier portfolios are normally held by wealthier households)» (Attanasio and Weber, 2010, p. 707).\(^{11}\)

Quadratic preferences also imply that the elasticity of intertemporal substitution is decreasing, which is another feature that makes its use unappealing: when the EIS is decreasing, poor consumers are willing to substitute consumption over time in response to interest rate changes more than rich consumers.\(^{12}\)

### 3.2. Exponential preferences: constant absolute risk aversion

Exponential utility has played an important role in the literature on the consumption Euler equation, being employed in early contributions investigating the relevance of a precautionary motive for saving, such as Kimball and Mankiw (1989) and Caballero (1990). In fact, it allows to go beyond certainty equivalence by retaining much of its analytical tractability.

For a consumer with exponential preferences, Euler equation (2) becomes:

\[ \exp\{-\alpha C_t\} = \left(\frac{1+r}{1+r}\right) E_t[\exp\{-\alpha C_{t+1}\}] \]

which reduces to

\(^{9}\) Notice that it is necessary to assume that the agent’s wealth is such that consumption is always in the range where marginal utility is positive.

\(^{10}\) See Blanchard and Mankiw (1988) and Caballero (1990) as early contributions expressing discontent with the certainty-equivalence model, and Browning and Lusardi (1996) for a thorough discussion.

\(^{11}\) Talking about the IARA hypothesis underlying quadratic utility, Blanchard and Mankiw state that «Introspection and casual evidence suggest that this is a poor description of behavior under uncertainty» (Blanchard and Mankiw, 1988, pp. 173–174).

\(^{12}\) Several empirical studies suggest that the EIS increases with consumption (see Section 4).
\[\mathbb{E}_t[C_{t+1}] = C_t + \ln\left(\frac{1+r}{1+\delta}\right)^{1/\alpha}\]  \hfill (6)

where the term \(\ln\left(\frac{1+r}{1+\delta}\right)^{1/\alpha}\) represents the intertemporal substitution motive for saving.

This functional form is unique in exhibiting constant absolute risk aversion, which implies that the elasticity of risky investments with respect to wealth is zero. In addition, since under CARA absolute risk aversion coincides with absolute prudence, the latter is constant as well. This implies that the increase in consumption required to keep the same level of expected marginal utility in the face of an increase in uncertainty is independent of the initial level of consumption, i.e., poor people and rich people reduce their consumption by exactly the same amount in reaction to a given risk. As Blundell and Stoker argue,

\[\text{his feature makes CARA preferences particularly inflexible, and inappropriate for empirical analysis. Even if the parameters of the CARA model were set to give a plausible value of precautionary savings in one period, in a multiperiod framework in which subsequent periods’ wealth is revised, the original parameter choice is unlikely to give plausible results (and Blundell and Stoker, 1999, p. 483).}\]

Moreover, exponential utility implies increasing relative risk aversion and decreasing intertemporal substitution, which are quite unrealistic assumptions as well. These features, together with the fact that exponential utility does not rule out negative consumption, make this functional form an improper representation of individual preferences.

### 3.3. Power utility: constant relative risk aversion

Power (or CRRA, or isoelastic) utility has been used in the Euler equation literature ever since the papers by Hansen and Singleton (1982, 1983) and is by far the most popular preference specification. This functional form is also the usual choice in empirical works aiming at estimating the coefficient of relative risk aversion, the intensity of the precautionary motive for saving, or the response to intertemporal incentives, because it leads to an approximation of the Euler equation which is linear in parameters. In fact, under isoelastic preferences, the Euler equation becomes

\[\mathbb{E}_t\left[\frac{C_{t+1}}{C_t}\right] = \left(\frac{1+r}{1+\delta}\right)^{1/\gamma}\]  \hfill (7)

from which the following testable relationship can be derived by taking logs and assuming that consumption is approximately log-normally distributed:

\[\Delta \ln C_{t+1} = \hat{\beta} + \frac{1}{\gamma} r_t + \frac{\gamma}{2} \text{Var}(\Delta \ln C_{t+1}) + u_{t+1}\]  \hfill (8)

with \(\mathbb{E}_t[u_{t+1}] = 0\).

\[13\] When marginal utility goes to infinity for consumption going to zero, this endogenously restricts optimal consumption within the positive range. Under exponential preferences, instead, the Euler equation needs to be combined with the non-negativity constraint on consumption.
In equation (8), \(\tilde{\beta} = \frac{1}{\gamma} \ln \left( \frac{1}{1 + \delta} \right)\) is the discount factor; \(\frac{1}{\gamma}\) measures the response to the interest rate and thus represents the elasticity of intertemporal substitution; and \(\text{Var}(\Delta \ln C_{t+1})\) is the variance of future consumption conditional on information available at time \(t\), so that \(\frac{\gamma}{2}\) captures the intensity of the precautionary motive for saving.

As apparent from equation (8), the curvature parameter \(\gamma\) plays a double role. On the one hand it is equal to the (constant) coefficient of relative risk aversion and therefore summarizes the consumer’s attitude towards risk; on the other, its reciprocal is equal to the (constant) EIS and therefore measures how consumption growth changes in response to changes in the relative price of future consumption. For that matter, specifying the degree of risk aversion also pins down the degree of prudence (see Table 2). For example, a coefficient of relative risk aversion of 5 would imply an EIS equal to 0.2 and a coefficient of relative prudence equal to 6, whereas logarithmic utility, which is obtained as a special case of isoelastic preferences by imposing \(\gamma = 1\), implies the coefficient of relative risk aversion and the EIS to be both equal to unity and the coefficient of relative prudence to be 2.

Basically, power utility falls short of flexibility in two regards: first, a single parameter governs risk aversion, intertemporal substitution, and prudence; second, the EIS is constrained, \textit{a priori}, to be independent of consumption, in contrast with the empirical evidence suggesting that rich agents are less averse to proportional fluctuations in consumption. These unappealing features of power utility are often completely neglected, despite several scholars having displayed their discontent. For example, Browning and Lusardi (1996) argue that «[it is not completely clear why [the usual choice of utility function is the power form] because the CRRA form has some shortcomings» (p. 1826). Crossley and Low (2011) discuss some of the significant implications of assuming a constant EIS and provide a test of this assumption, which is rejected. As a consequence of the dissatisfaction with the power form for preferences, some scholars prefer adopting EZW preferences, thereby relaxing the link between the EIS and risk aversion but maintaining the constant EIS hypothesis.


While quadratic, exponential, and isoelastic preferences represent different specifications of the instantaneous utility function within the expected utility framework, EZW preferences use a nonlinear aggregator to bring together present and future utility, thereby dropping the expected utility hypothesis. Indeed, under the specification devised by Epstein and Zin (1989) and Weil (1990), utility is recursively defined over current consumption and a certainty equivalent of future random utility. In a two-period framework, the consumer maximizes \(u(C_1) + \beta u(\hat{C}_2)\), where \(\hat{C}_2\) is the certainty equivalent level of consumption and satisfies \(V(\hat{C}_2) = E_t[V(C_2)]\), and \(u(\cdot)\) and \(V(\cdot)\) are concave functions. Usually, this functional form is defined so as to take advantage of the analytical tractability of the isoelastic specification, with the power form applied over both consumption in the current
period and the certainty equivalent in the next period (see Table 1 for the general functional form and Table A.1 for the simpler, two-period case).

With EZW preferences, there are two coefficients in the utility function and so the degree of risk aversion (γ), determined by the curvature of \( V(C) \), is disentangled from the EIS (σ), determined by the curvature of \( u(C) \). This provides a kind of flexibility which is precluded with different functional forms, allowing for a high aversion to risk and a high aversion to intertemporal substitution to coexist. However, since the isoelastic form is preserved, the parameters are both constrained to be constant, thereby retaining the criticism concerning an EIS independent of the level of consumption. In addition, the separation between the preference parameters governing attitudes towards time and state fluctuations of consumption comes at the cost of accepting that the agent has a preference for the timing of resolution of uncertainty: whenever the consumer has greater (smaller) aversion to risk than to intertemporal substitution, early (late) resolution of uncertainty is preferred. As a matter of fact, EZW preferences can be conceived as a generalization of the standard time additive expected utility function, which collapses to the isoelastic case when the coefficient of relative risk aversion equals the reciprocal of the EIS. Epstein and Zin (1989) show that EZW utility with \( σγ ≠ 1 \) implies that the temporal resolution of risk matters, and Epstein et al. (2014) provide a quantitative assessment of timing premia. The timing premium is defined as the fraction of the consumption stream that the agent would be willing to give up in order for all risk to be resolved, and depends on both the preference parameters and the nature of the endowment process: the farther the elasticity of intertemporal substitution from the reciprocal of relative risk aversion and the more persistent the consumption process, the greater the timing premium.

Basically, if the isoelastic specification entails that a single parameter governs three aspects of preferences, with EZW utility, two parameters control four aspects of preferences. In fact, once the EIS and the degree of risk aversion are set, prudence follows accordingly. Kimball and Weil (2009) provide a thorough discussion of how the links between preference parameters take form in the case of recursive preferences. In particular, in the most popular version of EZW preferences, in which both the time aggregating function and the function that synthesizes attitudes towards risk are of the isoelastic form, relative prudence is

\[
p(C_t) = γ(1 + σ).
\]  

(9)

Relationship (9) shows that under this functional form the precautionary saving motive is always stronger than risk aversion (regardless of the EIS) and, at the same time, holding risk preferences fixed, the extent to which prudence is greater than risk aversion increases with the EIS.

From relationship (9) it is also possible to compute the magnitude of all preference parameters once a couple of values are assigned to the coefficients of the isoelastic EZW function, γ and σ. For example, a coefficient of relative risk aversion equal to 5 associated with an EIS equal to 1 would imply a coefficient of relative prudence equal to 10.
One difficulty that arises when reviewing the empirical literature on preference parameters is that, because of the links between them, most empirical estimates are conducted in such a way that, while investigating a specific aspect of preferences, they also have implications for the size of the parameters that do not represent the focus of the research. For example, if an empirical contribution relying on isoelastic utility provides an estimated value for the EIS equal to 0.5, it implicitly conveys an estimation for relative risk aversion of 2. We hope that the arguments presented so far (and Table 2) allow the reader to derive such interrelations.

Since recent surveys of the empirical literature on each preference parameter are available, we briefly discuss the evidence based on Euler equation estimation, with a focus on the functional forms adopted.

The first estimates of preference parameters appeared in the finance literature and concerned the degree of risk aversion. Breeden (1979) developed the intertemporal model that has been widely employed in subsequent research, but the finance paper that has had the greatest influence is probably that by Hansen and Singleton (1983). Their estimates of the coefficient of relative risk aversion, relying on a power utility function, were between 0 and 2 with five out of six estimates being smaller than unity. In other early studies on U.S. data, all based on Hansen and Singleton’s framework and on CRRA utility, the estimates range between 0.5 (as in Mankiw et al., 1985) and 4 (as in Mankiw, 1981), which are considered plausible values. But Mehra and Prescott (1985), analysing the difference between the average return on the stock market and the return on short-term government debt in the U.S., find what is known as the equity premium puzzle: calibrating an asset pricing model with what they consider reasonable values of the parameters, including a coefficient of relative risk aversion for isoelastic preferences below or equal to 10, they are not able to produce the equity premium observed, which would require consumers to have an implausibly high aversion to risk, falling in the 20–30 range.\textsuperscript{14} Epstein and Zin (1991) employ EZW preferences and find estimates of relative risk aversion hovering around 1, but this specification of preferences does not seem to solve the puzzle.\textsuperscript{15} The subsequent research has adopted either power utility, possibly embedding habit formation, or EZW preferences, with extremely different estimates of relative risk aversion, ranging from below 1 to over 30. For a review of the empirical evidence on the magnitude of risk aversion, also in-

\textsuperscript{14} It might be useful to provide an example giving an insight into what can be considered a plausible range for the coefficient of relative risk aversion. Let us assume that an agent with isoelastic preferences is offered a gamble where he can win 100 or 200 units of consumption, with 50% probability attached to each outcome. Consider the level of consumption $X$ that would make the agent indifferent between accepting the gamble and receiving $X$ with certainty: with $\gamma$ varying between 0.5 and 4, $X$ ranges from 146 to 121, while if $\gamma = 20$, then $X = 104$.

\textsuperscript{15} See Kocherlakota (1990, 1996) for a critical discussion of the empirical results stemming from the separation of risk aversion and intertemporal substitution.
cluding laboratory experiments and, in general, methodologies which lie outside the standard literature on consumption based on the Euler equation, see Outreville (2014).

Overall, there appears to be little consensus regarding the magnitude of this parameter or the direction in which it changes as wealth increases, so as to consider still appropriate what Gollier stated some years ago:

It is quite surprising and disappointing for me that almost 40 years after the establishment of the concept of risk aversion by Pratt and Arrow, our profession has not been able to attain a consensus about the measurement of risk aversion (Gollier, 2001, pp. 424–425).

The attempts to quantify the importance of the precautionary motive for saving have been extensive as well. Most of the early empirical studies include some measure of risk in a linear approximation to a consumption Euler equation and test for its significance. These tests typically find that the estimated effects of consumption uncertainty on consumption growth are small, indicating that precautionary motives are weak or nonexistent. A very popular contribution is that by Dynan (1993), who examines lifetime saving behaviour in the U.S. and obtains extremely small and statistically insignificant estimates of relative prudence with CRRA utility. Ruehlewein (1991), Grossberg (1991), and Parker (1999) have all failed to obtain evidence of precautionary saving in the U.S. as well. Some of the later studies find greater estimates but, overall, there has been a wide discrepancy between the values of prudence assumed in simulations and those – very small – implied by estimations performed on actual savings data, which suggests a widespread belief in prudent attitudes and precautionary motives, and leads to the supposition that the empirical tests relying on Euler equation estimation failed to capture the true extent of prudence. Browning and Lusardi (1996) provide a detailed survey of the early contributions on precautionary saving, and Lugilde et al. (2017) review the recent evidence, which is just as ambiguous:

In spite of a rather large number of studies, empirical results are not conclusive. Most works find evidence of an effect of uncertainty on savings, but there is no consensus about the intensity of this reason for saving, nor on which is the most appropriate measure to approximate uncertainty (Lugilde, et al., 2017, p. 2).

As for the EIS, the approach to estimate it has been through the Euler equation for consumption applied first to aggregate time series, and, later, to household level data. An influential contribution is that by Hall (1988), who claims that, at least for the U.S., the EIS is close to zero. This result, obtained with a power specification for preferences, contradicts some of the earlier studies, such as Summers (1982) or Muellbauer (1983), which had provided estimates close to unity, and, most importantly, since $\alpha = 1/\gamma$, implies a coefficient of relative risk aversion which tends to infinity. However, many subsequent studies find very low estimates similar to Hall (1988).

As already mentioned, substantial evidence has been accumulated against the assumption of an EIS independent of consumption. Blundell et al. (1994) investigate the link be-
tween within-period and intertemporal allocation on U.K. micro data, adopting an isoelastic specification extended to allow for demographics, labour market status, and other household characteristics. They point out that the flexible specification of preferences, allowing the EIS to vary with consumption, leads to higher and better determined estimates, and conclude that «in terms of estimating the ISE it may be misleading to use an iso-elastic specification» (p. 71). The evidence provided by Attanasio and Weber (1995), Attanasio and Browning (1995), and Atkeson and Ogaki (1996) also suggests that the elasticity increases with consumption. More recently, Guvenen (2006) shows that introducing heterogeneity in the EIS, with larger values of the parameter for wealthier agents, improves the fit of a calibrated macroeconomic model. In addition, Crossley and Low (2011) provide a test based on the implications of the constant EIS hypothesis for the shape of Engel curves and conclude that the parameter cannot be the same for rich and poor households. All these findings suggest that isoelastic utility is ill-suited for performing estimates of the EIS, and yet it is still the functional form most employed. Always more frequently, starting with Epstein and Zin (1991), the expected-utility power form is replaced by EZW preferences, thereby relaxing the link between the EIS and risk aversion but maintaining the isoelastic form. Studies relying on EZW utility have sometimes obtained greater estimated values compared with those relying on the standard power function, such as those by Muligan (2002) and Choi et al. (2017), who found an EIS greater than unity for the U.S.

The contributions attempting to determine the EIS relying on Euler equation estimates are countless.\textsuperscript{16} The results of this strand of literature are dubious at best; there is still controversy about whether or not the EIS is large enough to make changes in expected interest rates an important factor in fluctuations in consumption growth. For that matter, Attanasio and Wakefield argue that

\[ \text{[d]espite the clear relevance of the EIS, a remarkable feature of the sizeable recent literature on the effect of the preferential taxation of retirement wealth on personal and national saving is that it never refers to the literature that has studied the life-cycle model and estimated preference parameters, including the EIS (Attanasio and Wakefield, 2010, p. 682).} \]

Moreover, there appears to be a contradiction between the econometric results and the dynamic macroeconomic literature, since calibrated general equilibrium models usually require an EIS close to one – or even greater – in order to replicate the evidence about growth and business cycles.\textsuperscript{17}

5. THE ROLE OF THE FUNCTIONAL FORM: A SIMULATION EXERCISE

In their thorough review of the early literature on the consumption Euler equation, Browning and Lusardi (1996) claim that the certainty-equivalence model can be very mis-

\textsuperscript{16} See Havránek et al. (2015) and Thimme (2017) for detailed accounts.

\textsuperscript{17} See the discussion in Guvenen (2006).
leading. To make their point, they present a numerical example which points out the crucial role of the hypothesis of quadratic preferences: by assuming that the utility function is logarithmic rather than quadratic, they show that the random walk result and all the main implications of the certainty-equivalence model drop.

This section, with the purpose of stressing the importance of the functional form adopted in the estimation of preference parameters, extends the numerical example proposed by Browning and Lusardi to the case of different utility functions and slightly more complicated hypotheses about the stochastic framework faced by the consumer. This simple simulation exercise is meant to show that the same saving behaviour can be associated with quite different attitudes towards time and uncertainty – that is, to different magnitudes of the preference parameters – depending on the utility function adopted. Obviously, such a simple two-period model like the one that will be presented does not allow for deriving general indications on the magnitude and the direction of the influence of the functional form on the outcome of an intertemporal optimizing problem. Nevertheless, it seems a straightforward way to suggest that the issue may deserve further investigation. Moreover, the simulation exercise will be associated with a more formal and general analysis of the impact of parameters’ values on the saving behaviour of the consumer, conducted by means of a decomposition of the rate of growth of consumption derived by the Euler equation.

5.1. Links to the simulation literature

Simulations of intertemporal maximization models have played an important role in the consumption literature of the last decades. Their importance stems from the fact that, except for very special cases, the Euler equation for consumption does not provide closed-form solutions, and so it does not allow for establishing how a change in the framework faced by the consumer affects his level of savings.

Early contributions employing simulation techniques are those by Skinner (1988), who attempts to quantify the precautionary component of aggregate savings, and Deaton (1991) and Carroll (1992), who solve buffer-stock models with prudent and impatient consumers. Also influential are the studies by Attanasio et al. (1999) and Gourinchas and Parker (2002), who rest on simulations to analyse the role of demographics in explaining the evidence of consumption tracking income. Attanasio and Wakefield (2010) simulate a life cycle model to assess the importance of the EIS in determining the size of the reaction of savings to changes in the interest rate. All these contributions adopt power utility. More recently, Choi et al. (2017) perform simulations which rely on EZW preferences to quantify the impact of varying the parameters on the implications of their model.

What distinguishes the simulations here presented with respect to previous literature is the replication of the exercise not only with different values assigned to the parameters, but also with different utility functions. In addition, the model of utility maximization em-
ployed is characterized by the intentionally simple framework considered. In this regard, it is important to stress that simulation results should generally be considered with caution. In fact, when simulating a model, one needs to make very specific modelling assumptions, choose a particular utility function, and assign a value to each of its parameters. As realistic as the simulated model can be, some aspects are bound to remain stylized, and, at the same time, the sensitivity to assumptions suggests that such results should be interpreted with prudence.\textsuperscript{18} However, the simulation exercise here presented does not seem to be affected by such general limitation of the approach. In fact, the purpose is not that of replicating actual paths of consumption across the life cycle of an agent, nor do we aim at giving a quantitative assessment of the impact of the functional form on preference parameters. On the contrary, the simulation exercise is supposed to show that, ceteris paribus, a different utility function leads to the same consumption behaviour being associated with very different values of the structural parameters. For this reason, the consideration of a very simple model, in which all other circumstances affecting the values of preference parameters are ruled out, seems particularly effective for our purposes.

5.2. The misleading role of quadratic preferences: a numerical example

In their two-period model, Browning and Lusardi (1996) consider the standard framework in which an agent has an intertemporally additive utility function, faces perfect capital markets, and maximizes expected utility forming rational expectations. They assume that the within-period utility function is logarithmic and that the rate of time preference and the rate of interest are both equal to zero, so that the agent maximizes $\ln C_1 + \mathbb{E}_1[\ln C_2]$. In the first period, the agent has a certain income $Y_1$, while second-period income is equal to zero with probability $\varepsilon$, and to $Y_2/(1-\varepsilon)$ with probability $(1-\varepsilon)$. Thus, the expected value of second-period income is $Y_2$, and an increase in $\varepsilon$ represents a mean-preserving increase in uncertainty.

Browning and Lusardi imagine two possible cases, which imply, respectively, low or high first-period income relative to second-period expected income. They first show that in the case of perfect certainty ($\varepsilon = 0$), regardless of the time profile of earnings, the agent consumes half of lifetime (expected) resources in each period. This is the same result one gets in the presence of uncertainty when preferences are quadratic: in both cases – under certainty and under certainty equivalence – the only motive for saving is the life-cycle motive, that is, the consumer saves in order to smooth the lifetime path of consumption. By contrast, when $\varepsilon > 0$ (in their example, $\varepsilon = 0.01$) and the agent exhibits nonquadratic preferences, the time profile of income becomes crucial in determining that of consump-

\textsuperscript{18} As stressed by Attanasio and Wakefield, «[...]he big difficulty of this approach, if it wants to be realistic and of policy relevance, is that one needs to specify all of the details of the stochastic environment in which the consumer lives. And, [...] some of these details are quantitatively and qualitatively important for the results one obtains» (Attanasio and Wakefield, 2010, p. 707).
tion. In the latter case, indeed, in addition to the life-cycle motive for saving, the agent has a precautionary motive triggered by uncertainty.

Assuming first $Y_1 = 2$ and $Y_2 = 1$ (high first-period income scenario) and then $Y_1 = 1$ and $Y_2 = 2$ (low first-period income scenario), they show that the degree to which the certainty-equivalence model approximates the model with logarithmic preferences depends on the time path of expected income. In particular, the certainty-equivalence model’s prediction that the marginal propensities to consume out of current and future income are the same is wildly wrong for agents with low first-period income.

This numerical example, though very simple, provides a clear insight into the crucial role that an inappropriate utility function can play. The explanation for such misleading results, in the case of quadratic preferences considered by Browning and Lusardi, lies in the linearity of marginal utility: as already argued (see Sections 2.2 and 3.1), it is only when marginal utility is convex that uncertainty has an impact on current consumption.

The explicative power of the numerical example just discussed suggests extending this simple model to the case of alternative utility functions in order to evaluate the impact of the functional form and of the values of preference parameters on saving behaviour.

5.3. A numerical example with different preference specifications

The first simulation we are going to discuss is a replication of the two-period model example proposed by Browning and Lusardi (1996) in which we adopt isoelastic and exponential preferences rather than logarithmic, retaining all the other assumptions: $\delta = r = 0$, first-period income is $Y_1$, and second-period earnings are equal to zero with probability $\varepsilon$ and to $Y_2/(1 - \varepsilon)$ with probability $1 - \varepsilon$, with $\varepsilon = 0.01$.

When utility is logarithmic, the coefficient of relative risk aversion and the EIS are equal to one and the coefficient of relative prudence is two (see Table 2). Conversely, once we leave the logarithmic specification, the degree of prudence (and risk aversion) is determined by the parameter of the function – $\alpha$ in the exponential preferences case and $\gamma$ when dealing with the isoelastic form. Assigning different values to the parameter of the utility function, chosen accordingly with the existing empirical literature, we obtain different saving behaviours. As displayed in Figure 1, the same saving rate is associated with quite different attitudes towards uncertainty depending on the utility function adopted.19

Consider the low first-period income scenario ($Y_1 = 1$ and $Y_2 = 2$): in the case of certainty, or certainty equivalence, first-period consumption would be $C_1 = 1.5$ regardless of the degree of risk aversion. Conversely, when the agent exhibits prudence, first-period consumption decreases as risk aversion increases. However, the pace at which savings rise with risk aversion depends on the utility function adopted, being higher in the case of isoelastic preferences than under exponential utility. Therefore, a precautionary motive for

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19 Notice that the intertemporal motive for saving is ruled out here because of the null rate of interest, so that the saving behaviour depends entirely on the time profile of earnings and on risk preferences.
saving of the same intensity is associated with substantially different behaviours under the two preferences’ specifications. For example, imagine we observe an agent consuming 0.855, which implies a saving rate equal to 0.145. If we assume an isoelastic utility function, we shall conclude that the agent displays a degree of relative prudence equal to 3.54, while if we adopt exponential preferences, we shall deduce that relative prudence is equal to 5.55. That also means that the corresponding coefficient of relative risk aversion ranges from 2.54 (under isoelastic utility) to 5.55 (with exponential preferences). Such ambiguity shows that the role of the functional form adopted in the estimation of preference parameters through the Euler equation is crucial.

5.4. The relationship between preference parameters and the saving rate

Before proceeding to the discussion of a more sophisticated simulation exercise, it is useful to dwell on a formal and general analysis of the impact of parameters’ values on the saving behaviour of the consumer, which helps interpreting the results presented in the next section. In fact, even though closed-form solutions for consumption are generally not available for models of intertemporal utility maximization, it is possible to characterize analytically the relationship between preference parameters and the saving rate drawing on previous contributions which deduce some of the properties of the optimal behaviour of the consumer from an analysis of the Euler equation. The purpose is to infer how a change in the parameters of the utility function affects the saving rate in each of the functional
forms employed in the simulations to be presented: exponential, isoelastic, and recursive EZW preferences.

In the case of exponential preferences, the first-order conditions of the optimization problem of the consumer give equation (6), which shows that, under CARA utility, intertemporal substitution does not take a multiplicative form that implies changes affecting the rate of growth of consumption, but entails additive changes in the level of consumption. Under suitable hypotheses, Caballero (1990) derives a closed-form solution for consumption in the case of exponential utility and shows that this functional form implies a consumption function that, when \( \delta = r \), may be decomposed, additively, in a term analogous to that of the certainty-equivalence case and a precautionary saving component. This means that for any level of wealth, consumption is equal to consumption under quadratic preferences minus a term related to precautionary saving behaviour. If \( \delta \neq r \), it is still possible to decompose the consumption function into a precautionary saving term and a second term but the latter is no longer identical to consumption under certainty equivalence. Caballero also shows that the slope of the consumption path when \( \delta = r \) is positive.

In the case of power utility or isoelastic EZW preferences, from the log-linearized version of the Euler equation it is possible to infer the variation of the expected rate of growth of consumption \( \mathbb{E}(\Delta \ln C_{t+1}) \), interpreted as a proxy for the saving rate, in response to a change in the parameters of the utility function. For the case of power utility, Parker and Preston (2005) use the Euler equation to decompose consumption growth into its different sources and, applying such decomposition to households’ expenditure data, they measure the relative importance of each source in determining fluctuations in average consumption growth. As for EZW preferences, Choi et al. (2017), rely on a decomposition of the rate of growth of consumption to distinguish between precautionary and nonprecautionary components.

Under the assumption of isoelastic preferences, the Euler equation takes the form of equation (7), from which, by assuming that consumption is log-normally distributed, one gets the log-linearized version of the Euler equation:

\[
\mathbb{E}(\Delta \ln C_{t+1}) = \frac{r-\delta}{\gamma} + \frac{1}{2} \text{Var}(\Delta \ln C_{t+1}).
\]

(10)

Equation (10) allows to disentangle the intertemporal motive for saving, summarized by the coefficient on \((r - \delta)\), which represents the EIS, from the precautionary motive, measured by the coefficient on the variance of future consumption. The isoelastic specification, though, implies that both components depend on the same coefficient \(\gamma\).

Differentiating (10) with respect to \(\gamma\) gives

\[
\frac{\partial \mathbb{E}(\Delta \ln C_{t+1})}{\partial \gamma} = \left(\frac{\delta - r}{\gamma^2}\right) + \frac{1}{2} \text{Var}(\Delta \ln C_{t+1}) + \frac{\gamma \partial \text{Var}(\Delta \ln C_{t+1})}{\partial \gamma}.
\]

(11)

Equation (11) shows that the impact of a change in \(\gamma\) on the saving behaviour is ambiguous. An increase in risk aversion has indeed a positive effect on the rate of growth of consumption for “impatient” agents, namely when \(\delta > r\), plus a negative effect through the
impact on the variance of consumption. In fact, if $\gamma$ rises, the EIS decreases, which means that the consumer is more averse to intertemporal fluctuations of consumption and is inclined to lower consumption volatility. The total effect of an increase in $\gamma$ on the saving rate of an impatient consumer is positive as long as the indirect effect on the variance is not so high as to offset the direct effect.

The analysis of the impact of a change in preference parameters becomes more complex under the hypothesis of EZW utility, since it implies a distinction between risk aversion and the EIS, both of which affect the rate of growth of consumption.\(^{20}\)

With EZW preferences, the Euler equation takes the form

$$1 = \frac{1+r}{1+\delta} E_t \left[ \frac{C_{t+1}}{C_t} \frac{(\delta)^{\frac{1}{\gamma}}}{\left(\mathbb{E}[V_{t+1}^{1-\gamma}]\right)^{1-\gamma}} \right]. \quad (12)$$

Following Choi et al. (2014), we let $Z_{t+1}$ be next-period utility relative to its certainty equivalent and refer to it as a preference shifter:

$$Z_{t+1} \equiv \frac{V_{t+1}}{(\mathbb{E}[V_{t+1}^{1-\gamma}])^{1-\gamma}}$$

With this notation, we can write the Euler equation as

$$1 = \frac{1+r}{1+\delta} E_t \left[ \frac{C_{t+1}}{C_t} \frac{1}{\gamma (1-\gamma)} Z_{t+1} \right],$$

whose log-linearized version is:

$$\mathbb{E}[\Delta \ln C_{t+1} - (1 - \gamma \sigma) \ln Z_{t+1}] = \sigma \left( r - \delta \right) + \frac{1}{2\sigma} \mathbb{V}[\Delta \ln C_{t+1} - (1 - \gamma \sigma) \ln Z_{t+1}], \quad (13)$$

which can be rearranged as:

$$\mathbb{E}[\Delta \ln C_{t+1}] = \sigma \left( r - \delta \right) + (1 - \gamma \sigma) \mathbb{E}(\ln Z_{t+1}) + \frac{1}{2\sigma} \mathbb{V}[\Delta \ln C_{t+1} - (1 - \gamma \sigma) \ln Z_{t+1}] \quad (14)$$

The first term in (14) is the intertemporal substitution effect, which depends on the gap between the interest rate and the subjective rate of time preference as well as on the EIS. The second term represents the preference for the timing of resolution of uncertainty, which depends on $\gamma \sigma$: if $\gamma < 1/\sigma$, then the term is positive since the consumer prefers later resolution of uncertainty, which raises saving.\(^{21}\) The last term is the precautionary effect, which increases with consumption volatility and whose intensity depends on both risk aversion and the EIS, since $p(C) = \gamma + \gamma \sigma$ under EZW preferences.

Differentiating equation (14) with respect to $\gamma$ and $\sigma$ shows that the impact of a change in parameters on the saving behaviour is ambiguous: both the EIS and risk aversion have a nonmonotonic relationship with the rate of growth of consumption.

\(^{20}\) The analytical characterization of the impact of a change in parameters on the rate of growth of consumption for the case of EZW preferences can be found in Choi et al (2014), the working paper version of the contribution published in 2017. Here we just summarize their main results.

\(^{21}\) Notice that when $\gamma \sigma = 1$ the timing of resolution of uncertainty becomes irrelevant and, consistently, the preference shifter goes to zero and equation (14) reduces to (10), the log-linearized Euler equation for power utility.
As for changes in the EIS, if \( \delta > r \), then the intertemporal substitution effect is negative. In fact, an increase in \( \sigma \) lowers the aversion to consumption fluctuations and leads an impatient agent to anticipate consumption to the current period. The effect of increasing the EIS on the preference shifter is negative as well, if we suppose that risk aversion and intertemporal substitution are high (\( \gamma > 1 \) and \( \gamma \sigma > 1 \)). In this case, raising \( \sigma \) results in a stronger preference for early resolution of uncertainty, which lowers the preference shifter, and, at the same time, implies a smaller aversion to fluctuations of consumption which results in higher volatility. Finally, the effect of increasing the EIS on the precautionary component is twofold. On the one hand, lowering the aversion to temporal fluctuations of consumption reduces precautionary saving; on the other, a greater \( \sigma \) has, when \( \gamma \sigma > 1 \), a positive impact on the variance of consumption through its effect on the preference shifter. An increase in volatility raises precautionary saving, thus the two effects have opposite signs for consumers with a preference for early resolution of uncertainty.

The overall relationship between the EIS and the rate of growth of consumption cannot be unambiguously signed:

\[
\frac{\partial \Delta \ln C_{t+1}}{\partial \sigma} = (r - \delta) + \frac{1}{2} \text{Var}(\ln Z_{t+1}) - \frac{1}{2\sigma^2} \text{Var}(\ln Z_{t+1} - \Delta \ln C_{t+1}) + \frac{1}{\sigma} \left\{ \frac{\partial \text{Var}(\Delta \ln C_{t+1})}{\partial \sigma} + (\gamma \sigma - 1)(\sigma - 1) \frac{\partial \text{Var}(\Delta \ln Z_{t+1})}{\partial \sigma} + 2(\gamma \sigma - 1) \frac{\partial \text{Cov}(\Delta \ln C_{t+1}, \ln Z_{t+1})}{\partial \sigma} \right\}.
\]

(15)

The impact of a change in risk aversion is also nonmonotonic. In terms of the decomposition (14), discussed by Choi et al. (2014), risk aversion has no effect on the intertemporal substitution effect, but affects the preference shifter and the precautionary component. As for the former, when \( \gamma \sigma > 1 \), an increase in \( \gamma \) increases the desire for early resolution of uncertainty thereby lowering saving, as long as risk aversion is relatively high. On the contrary, when risk aversion is low, increasing \( \gamma \) raises the preference shifter and hence savings. At the same time, an increase in risk aversion lowers consumption volatility, which has a positive impact on the preference shifter for \( \gamma \sigma > 1 \) and \( \gamma > 1 \).

The impact of increasing risk aversion on the precautionary component arises as a result of different effects. First, an increase in \( \gamma \) affects the contribution of the preference shifter to volatility: if \( \gamma \sigma > 1 \), then the preference shifter has a positive effect on the variance of consumption thereby reinforcing the precautionary motive. At the same time, the overall variability of consumption is expected to fall with \( \gamma \), thus lowering precautionary saving.

As a consequence, just like the EIS, risk aversion has an ambiguous impact on the rate of growth of consumption:

\[
\frac{\partial \Delta \ln C_{t+1}}{\partial \sigma} = \left( \frac{\sigma - 1}{2} \right) \text{Var}(\ln Z_{t+1}) + \text{Cov}(\Delta \ln C_{t+1}, \ln Z_{t+1}) + \frac{1}{\sigma} \left\{ \frac{\partial \text{Var}(\Delta \ln C_{t+1})}{\partial \gamma} + (\gamma \sigma - 1)(\sigma - 1) \frac{\partial \text{Var}(\Delta \ln Z_{t+1})}{\partial \gamma} + 2(\gamma \sigma - 1) \frac{\partial \text{Cov}(\Delta \ln C_{t+1}, \ln Z_{t+1})}{\partial \gamma} \right\}.
\]

(16)

In conclusion, the relationship between the value of each preference parameter and consumption behaviour is inherently complex, changing with the level of lifetime resources, the degree of uncertainty, and, ultimately, with the overall definition of the sto-
chastic framework faced by the consumer. Such relationship, as the above analysis highlights, also depends on the functional form adopted for representing preferences, in a way which cannot be unambiguously identified and isolated from other sources of complexity.

5.5. The interplay of precautionary and intertemporal motives in a two-period model

In this section, the results of a slightly more sophisticated model are presented. First of all, in the example proposed by Browning and Lusardi (1996) there is a positive (though very little) chance that second-period income may be null. This is not a problem for the purpose of their argument, but in the context here considered such a strong hypothesis could crucially affect the results. In fact, when the utility function implies that marginal utility goes to infinity as consumption goes to zero, then if null income is a non-zero probability event, the consumer will never want to borrow. As a consequence, a first, simple extension of the baseline model will consist in assuming a positive second-period income also in the worst case scenario. It is supposed that in the second period earnings are $Y_2/2$ with probability $\varepsilon$ and $(2-\varepsilon)Y_2/2(1-\varepsilon)$ otherwise; thus expected second-period income is still equal to $Y_2$ as in the previous model.

Evidently, as long as the assumptions of a zero rate of time preference and a zero interest rate are retained, we can evaluate the interplay of life-cycle and precautionary motives in determining the saving behaviour of the consumer, but the intertemporal substitution motive is ruled out. For this reason, the model is extended to the case of a positive interest rate. In particular, the simulations consider the case of an impatient consumer in the sense of Carroll (1997), which implies that the subjective rate of time preference is greater than the rate of interest, so that the agent has an incentive to anticipate consumption from period 2 to period 1. The values assigned to the discount factor $\beta = (1 + \delta)^{-1}$ and to the interest factor $R = 1 + r$ are, respectively, 0.96 and 1.02, so that $\beta r = 0.98$. These values are consistent with those encountered in the simulation literature. Finally, a slightly higher degree of uncertainty seems more appropriate for the analysis of the relations between consumption profiles and preference parameters, hence $\varepsilon$ is set at 5%.

In this more sophisticated exercise, the analysis is applied to the more general EZW functional form in addition to the CARA and CRRA functions. The preference specification and the equilibrium condition for the two-period EZW utility are presented in Table A.1. For the case of EZW preferences, in which both the coefficient of relative risk aversion and the EIS can change, the simulations are performed for $\gamma = 1$ and a variable EIS and for $\sigma = 1$ and a variable risk aversion (with both cases collapsing to the logarithmic scenario when $\gamma \sigma = 1$).

---

22 This happens under power utility, whereas exponential preferences imply $u'(0) = 1$.

23 An interest factor $R = 1.02$ is adopted, for example, in the baseline model in Attanasio and Wakefield (2010), and both Guvenen (2006) and Carroll (2011) set $\beta = 0.96$. 
The results of the simulations are summarized in the Appendix, where tables are presented for each utility function and each scenario (low or high first-period income). For reasonable values of preference parameters, the corresponding level of first-period consumption, saving rate, expected value of second-period consumption, and variance of second-period log consumption are reported.

As a benchmark case, let us first consider how the consumer would behave in the case of quadratic preferences that rule out the effect of uncertainty. In such a scenario, consumption exhibits a decreasing profile because of the hypothesis on the value of βR: since the agent is impatient, he allocates a larger portion of lifetime resources to the current relative to the future period. This happens regardless of the degree of uncertainty on future income and of the ratio between current and future expected earnings, because of the certainty equivalent character of the model with quadratic preferences. In particular, the consumer would set first-period consumption such that
\[ C_1 = \frac{V_1}{1+\beta} + \frac{V_2}{R(1+\beta)}. \]

Therefore, the agent would consume 1.51 in the first period, which implies an expected value of future consumption equal to 1.48.

Once we leave the special case of quadratic preferences, first-period consumption depends upon the value of preference parameters.

Let us consider how consumption behaviour changes with changes in the EIS, by referring to Figure 2 which plots the values of \( C_1 \) when the elasticity varies between 0.1 and 1.5, the range into which almost all empirical estimates fall. First of all, the variability of current consumption in response to changes in the EIS depends on the ratio between first-period income and second-period expected income. This happens because in the case of high first-period income the consumer is a net saver.

When the EIS increases, the agent is less averse to intertemporal fluctuations of consumption and, at the same time, in the exponential and isoelastic cases, aversion to risk decreases. For CRRA utility, the decomposition of the saving rate \( (11) \) shows that a rise in the EIS has a negative direct effect on the saving rate when the consumer is impatient, and a positive indirect effect through the variance of second-period consumption. The overall effect on saving is negative as long as the effect operating through consumption variability is not too high, and thus causes current consumption to rise.24

In the case of EZW preferences, varying the EIS does not entail any changes in the degree of risk aversion, so that we can observe how consumption behaviour changes by keeping \( \gamma \) constant. In this case, when the aversion to intertemporal fluctuations in income decreases, there are an intertemporal substitution effect and a precautionary saving effect on the rate of growth of consumption. In our model, consumption declines when the EIS goes up in the case of low first-period income and vice versa.

24 The simulations are conducted for values of the EIS up to 4 and \( C_1 \) is monotonically increasing in \( \sigma \).
On the whole, the simulations performed show that the impact of the value of the preference parameters on consumption behaviour depends crucially upon the specification adopted for the utility function.

6. CONCLUSIONS

The estimation of the parameters of the utility function based on the Euler equation has engaged researchers over the last thirty years and has produced a great amount of empirical evidence, often rather conflicting. On the other hand, reaching an agreement on the values considered reasonable for preference parameters has become increasingly important for the theoretical and practical implications of the theory based on intertemporal utility maximization and for the calibration of general equilibrium models. In the light of the essential role of the values assigned to preference parameters, especially for devising and implementing policy measures, it seems important to make a general assessment of the results of the empirical evidence accumulated on the magnitude of the parameters. This pa-
per contributes to this purpose, focusing on the role of preferences specification. In fact, while several problematic issues affecting the results of the empirical research have been pointed out,\textsuperscript{25} the implications of adopting a specific utility function appear to have received little attention. \textbf{Browning and Lusardi (1996)}, when dealing with the «issues for modeling Euler equations» (p. 1825), also discuss «the choice of functional form» and, complaining about the widespread use of CRRA utility, argue that

\[ \text{among the criteria that should guide the choice of functional form are: congruence with theory (we should be able to impose theoretical restrictions in a simple way); flexibility (for example, important elasticities should not be constrained to be constant); and econometric tractability (for example, linearity in parameters is desirable) (Browning and Lusardi, 1996, p. 1826).} \]

An explicit reference to the issue can also be found in \textbf{Blundell and Stoker (1999)}, who regard power utility as the best choice for modelling saving behaviour and, after having expressed their reservations concerning quadratic and exponential preferences, conclude:

\[ \text{However, the main point is that the depiction of precautionary behavior is particularly sensitive to the choice of preferences, and so deserves explicit attention in model design (Blundell and Stoker, 1999, p. 503).} \]

Nevertheless, the issue seems to have been neglected later, when the only concern which can (sometimes) be found seems to be the one regarding the separation of risk aversion and intertemporal substitution. In fact, it could be argued that the choice of the functional form is usually made regardless of the features which make that function a proper representation of individual preferences: unappealing implications in terms of the direction in which the parameters change when consumption increases seem to be usually ruled out from the criteria of choice of the utility function. Isoelastic utility, which is definitely the most popular function, is consistent with the wildly shared hypothesis of decreasing absolute risk aversion, but it appears to have as its main advantage analytic convenience, since it yields first-order conditions that are log-linear in consumption. In fact, such a specification imposes strong restrictions on preferences as it has only one coefficient which must control three parameters and implies that relative risk aversion is the reciprocal of the elasticity of intertemporal substitution. In addition, the constant elasticity hypothesis that power utility underpins is incompatible with the evidence suggesting that wealthier households are more inclined to intertemporal substitution.

The analysis conducted shows that the mathematical definition of preference parameters implies quantitative constraints linking them to one another, which make the specification of the utility function crucial in assessing their values. It is in fact highlighted that different utility functions imply different links between the parameters and different hy-

\textsuperscript{25} Early influential contributions criticizing the common practice of employing the log-linearized consumption Euler equation for estimating preference parameters are those by \textbf{Carroll (2001)} and \textbf{Ludvigson and Paxson (2001)}. For recent contributions about the econometric issues related to preference parameters estimation, see \textbf{Alan and Browning (2010)} and \textbf{Alan et al. (2012)}.
hypotheses about the behaviour of preference parameters as consumption changes. In order to highlight the importance of the functional form adopted, a number of simulation exercises have been presented and discussed in the light of a decomposition of the rate of growth of consumption derived by the consumer’s Euler equation. The results show that the same consumption path can arise as a result of very different assumptions about the value of preference parameters depending on the utility function adopted. Moreover, the functional form is crucial in determining the effect of a change in one parameter on both the saving behaviour and the other parameters. At the same time, none of the utility functions usually adopted in the estimation of preference parameters appears completely satisfying. Quite the contrary, every functional form implies unappealing hypotheses on preferences, which clash with empirical evidence or common sense. As a result, the empirical work devoted to the estimation of preference parameters appears to have been substantially affected by the necessity of specifying a functional form for preferences, and the particular utility functions chosen may have played a fundamental role in determining the results obtained. Such remarks suggest that the line of research devoted to estimating preference parameters should consider more carefully the constraints imposed, on the one hand, by the quantitative definition of parameters, and, on the other, by the utility functions adopted.

7. REFERENCES


8. Appendix: Simulation results

Table A.1. Recursive EZW preferences in the two-period case

\[ V_t = u(C_t) + \beta u \left( \frac{1}{1-\rho} V_{t+1} \right) \]

where: \( u(C) = \frac{C^{1-\rho}}{1-\rho} \quad V(C) = \frac{C^{1-\gamma}}{1-\gamma} \quad \rho = \frac{1}{\delta} \)

<table>
<thead>
<tr>
<th>Utility function</th>
<th>Euler equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma \neq 1 \land \gamma \neq 1 )</td>
<td>( V_1 = \frac{C_1^{1-\rho}}{1-\rho} + \beta \frac{1}{1-\rho} \left[ E_1 \left( C_2^{1-\gamma} \right) \right]^{\frac{1-\rho}{1-\gamma}} )</td>
</tr>
<tr>
<td>( \sigma = 1 \land \gamma \neq 1 )</td>
<td>( V_1 = \ln C_1 + \beta \ln \left( E_1 \left( C_2^{1-\gamma} \right) \right) )</td>
</tr>
<tr>
<td>( \sigma \neq 1 \land \gamma = 1 )</td>
<td>( V_1 = \frac{C_1^{1-\rho}}{1-\rho} + \beta \frac{1}{1-\rho} \left[ e^{E_1 \left( ln C_2 \right)} \right]^{1-\rho} )</td>
</tr>
<tr>
<td>( \sigma = 1 \land \gamma = 1 )</td>
<td>( V_1 = \ln C_1 + \beta E_1 \left( \ln C_2 \right) )</td>
</tr>
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Table A.2

Exponential preferences low \( Y_t, \varepsilon = 0.05, \beta R = 0.98 \)

<table>
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<th>A(C_1)</th>
<th>a(C_1)</th>
<th>( \sigma(C_1) )</th>
<th>p(C_1)</th>
<th>P(C_1)</th>
<th>C_1</th>
<th>Savings rate</th>
<th>E(C_2)</th>
<th>Var(lnC_2)</th>
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<td>1.544</td>
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<td>1.65</td>
<td>0.61</td>
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<td>1.515</td>
<td>-51.5%</td>
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<td>-50.5%</td>
<td>1.487</td>
<td>0.063</td>
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<td>-49.5%</td>
<td>1.495</td>
<td>0.062</td>
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<td>1.49</td>
<td>1.000</td>
<td>1.487</td>
<td>-48.7%</td>
<td>1.503</td>
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</tr>
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<td>0.34</td>
<td>2.90</td>
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<td>-45.0%</td>
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Table A.3

Exponential preferences high \( Y_t, \varepsilon = 0.05, \beta R = 0.98 \)

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<tr>
<th>A(C_1)</th>
<th>a(C_1)</th>
<th>( \sigma(C_1) )</th>
<th>p(C_1)</th>
<th>P(C_1)</th>
<th>C_1</th>
<th>Savings rate</th>
<th>E(C_2)</th>
<th>Var(lnC_2)</th>
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Table A.4

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Table A.5

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<th>$P(C_i)$</th>
<th>$C_i$</th>
<th>Saving rate</th>
<th>$E(C_2)$</th>
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<td>2.00</td>
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<td>0.009</td>
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### Table A.6

EZW preferences RRA = 1 and variable EIS low \( Y_1, \varepsilon = 0.05 \) BR = 0.98

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<th>( p(C_1) )</th>
<th>( P(C_1) )</th>
<th>( C_1 )</th>
<th>Saving rate</th>
<th>( E(C_2) )</th>
<th>( \text{Var}[\ln C_2] )</th>
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<td>0.059</td>
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<td>1.50</td>
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<td>1.469</td>
<td>-46.9%</td>
<td>1.522</td>
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<td>1.90</td>
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<td>1.526</td>
<td>0.057</td>
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<td>2.20</td>
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### Table A.7

EZW preferences RRA = 1 and variable EIS high \( Y_1, \varepsilon = 0.05 \) BR = 0.98

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<th>( P(C_1) )</th>
<th>( C_1 )</th>
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<th>( E(C_2) )</th>
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<td>2.00</td>
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### Table A.8

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<th>P(C_1)</th>
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### Table A.9

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